Chapter 7

Right Triangles and Trigonometry

What You'll Learn

- **Lessons 7-1, 7-2, and 7-3** Solve problems using the geometric mean, the Pythagorean Theorem, and its converse.
- **Lessons 7-4 and 7-5** Use trigonometric ratios to solve right triangle problems.
- **Lessons 7-6 and 7-7** Solve triangles using the Law of Sines and the Law of Cosines.

Key Vocabulary

- geometric mean (p. 342)
- Pythagorean triple (p. 352)
- trigonometric ratio (p. 364)
- Law of Sines (p. 377)
- Law of Cosines (p. 385)

Why It's Important

Trigonometry is used to find the measures of the sides and angles of triangles. These ratios are frequently used in real-world applications such as architecture, aviation, and surveying. *You will learn how surveyors use trigonometry in Lesson 7-6.*

CONTENTS

Getting Started

Prerequisite Skills

To be successful in this chapter, you'll need to master these skills and be able to apply them in problem-solving situations. Review these skills before beginning Chapter 7.

For Lesson 7-1 — Proportions

Solve each proportion. Round to the nearest hundredth, if necessary. *(For review, see Lesson 6-1.)*

1. $\frac{3}{4} = \frac{12}{a}$ **2.** $\frac{c}{5} = \frac{8}{3}$ **3.** $\frac{e}{20} = \frac{6}{5} = \frac{f}{10}$ **4.** $\frac{4}{3} = \frac{6}{y} = \frac{1}{z}$

For Lesson 7-2 — Pythagorean Theorem

Find the measure of the hypotenuse of each right triangle having legs with the given measures. Round to the nearest hundredth, if necessary. *(For review, see Lesson 1-3.)*

5. 5 and 12 **6.** 6 and 8 **7.** 15 and 15 **8.** 14 and 27

For Lessons 7-3 and 7-4 — Radical Expressions

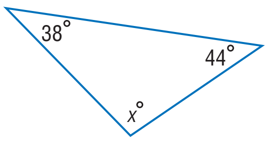
Simplify each expression. *(For review, see pages 744 and 745.)*

9. $\sqrt{8}$ **10.** $\sqrt{10^2 - 5^2}$ **11.** $\sqrt{39^2 - 36^2}$ **12.** $\frac{7}{\sqrt{2}}$

For Lessons 7-5 through 7-7 — Angle Sum Theorem

Find x. *(For review, see Lesson 4-2.)*

13.



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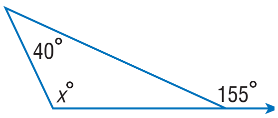

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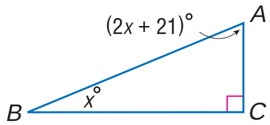
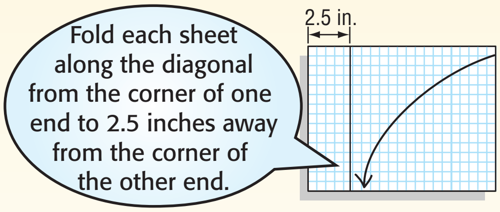


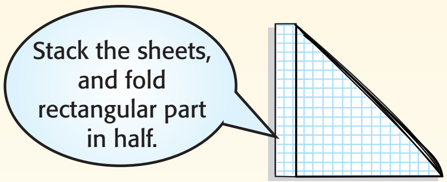
FOLDABLES™ Study Organizer

Right Triangles and Trigonometry Make this Foldable to record information from this chapter. Begin with seven sheets of grid paper.

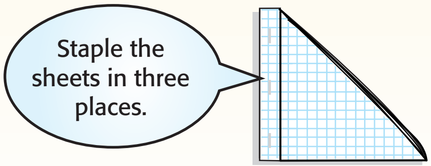
Step 1 Fold



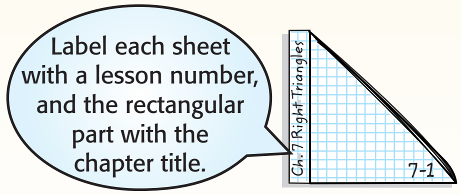
Step 2 Stack



Step 3 Staple



Step 4 Label



Reading and Writing As you read and study the chapter, write notes, define terms, and solve problems in your Foldable.

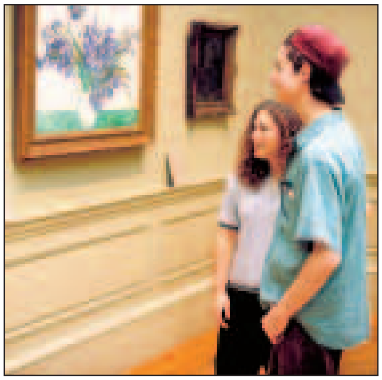
7-1 Geometric Mean

What You'll Learn

- Find the geometric mean between two numbers.
- Solve problems involving relationships between parts of a right triangle and the altitude to its hypotenuse.

How can the geometric mean be used to view paintings?

When you look at a painting, you should stand at a distance that allows you to see all of the details in the painting. The distance that creates the best view is the geometric mean of the distance from the top of the painting to eye level and the distance from the bottom of the painting to eye level.



Vocabulary

- geometric mean

GEOMETRIC MEAN The **geometric mean** between two numbers is the positive square root of their product.

Study Tip

Means and Extremes

In the equation $x^2 = ab$, the two x's in x^2 represent the *means*, and a and b represent the *extremes* of the proportion.

Key Concept — Geometric Mean

For two positive numbers a and b, the geometric mean is the positive number x where the proportion $a : x = x : b$ is true. This proportion can be written using fractions as $\frac{a}{x} = \frac{x}{b}$ or with cross products as $x^2 = ab$ or $x = \sqrt{ab}$.

Example 1 Geometric Mean

Find the geometric mean between each pair of numbers.

a. 4 and 9

Let x represent the geometric mean.

$\frac{4}{x} = \frac{x}{9}$ Definition of geometric mean

$x^2 = 36$ Cross products

$x = \sqrt{36}$ Take the positive square root of each side.

$x = 6$ Simplify.

b. 6 and 15

$\frac{6}{x} = \frac{x}{15}$ Definition of geometric mean

$x^2 = 90$ Cross products

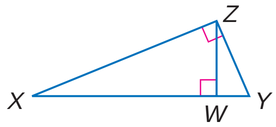
$x = \sqrt{90}$ Take the positive square root of each side.

$x = 3\sqrt{10}$ Simplify.

$x \approx 9.5$ Use a calculator.

Robert Brenner/PhotoEdit

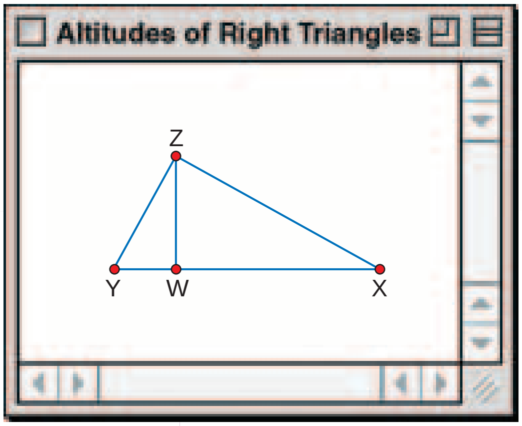
ALTITUDE OF A TRIANGLE Consider right triangle XYZ with altitude $\overline{WZ}$ drawn from the right angle Z to the hypotenuse $\overline{XY}$. A special relationship exists for the three right triangles, $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$.



Geometry Software Investigation

Right Triangles Formed by the Altitude

Use The Geometer's Sketchpad to draw a right triangle XYZ with right angle Z. Draw the altitude $\overline{ZW}$ from the right angle to the hypotenuse. Explore the relationships among the three right triangles formed.



Think and Discuss

1. Find the measures of $\angle X$, $\angle XZY$, $\angle Y$, $\angle XWZ$, $\angle XZW$, $\angle YWZ$, and $\angle YZW$.
2. What is the relationship between the measures of $\angle X$ and $\angle YZW$? What is the relationship between the measures of $\angle Y$ and $\angle XZW$?
3. Drag point Z to another position. Describe the relationship between the measures of $\angle X$ and $\angle YZW$ and between the measures of $\angle Y$ and $\angle XZW$.
4. Make a conjecture about $\triangle XYZ$, $\triangle XZW$, and $\triangle ZYW$.

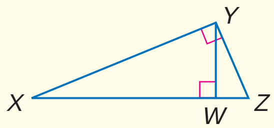
Study Tip

Altitudes of a Right Triangle
The altitude drawn to the hypotenuse originates from the right angle. The other two altitudes of a right triangle are the legs.

The results of the Geometry Software Investigation suggest the following theorem.

Theorem 7.1

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the two triangles formed are similar to the given triangle and to each other.



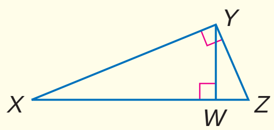
Example: $\triangle XYZ \sim \triangle XWY \sim \triangle YWZ$

You will prove this theorem in Exercise 45.

By Theorem 7.1, since $\triangle XWY \sim \triangle YWZ$, the corresponding sides are proportional. Thus, $\frac{XW}{YW} = \frac{YW}{ZW}$. Notice that $\overline{XW}$ and $\overline{ZW}$ are segments of the hypotenuse of the largest triangle.

Theorem 7.2

The measure of an altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the measures of the two segments of the hypotenuse.

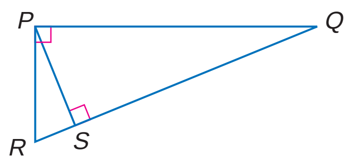


Example: YW is the geometric mean of XW and ZW.

You will prove this theorem in Exercise 46.

Example 2 Altitude and Segments of the Hypotenuse

In △*PQR*, *RS* = 3 and *QS* = 14. Find *PS*.



Let $x = PS$.

$\frac{RS}{PS} = \frac{PS}{QS}$

$\frac{3}{x} = \frac{x}{14}$ — $RS = 3$, $QS = 14$, and $PS = x$

$x^2 = 42$ — Cross products

$x = \sqrt{42}$ — Take the positive square root of each side.

$x \approx 6.5$ — Use a calculator.

PS is about 6.5.

Study Tip

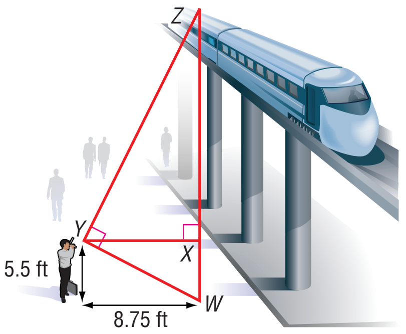
Square Roots
Since these numbers represent measures, you can ignore the negative square root value.

Ratios in right triangles can be used to solve problems.

Example 3 Altitude and Length of the Hypotenuse

ARCHITECTURE **Mr. Martinez is designing a walkway that must pass over an elevated train. To find the height of the elevated train, he holds a carpenter's square at eye level and sights along the edges from the street to the top of the train. If Mr. Martinez's eye level is 5.5 feet above the street and he is 8.75 feet from the train, find the distance from the street to the top of the train. Round to the nearest tenth.**

Draw a diagram. Let $\overline{YX}$ be the altitude drawn from the right angle of △*WYZ*.



$\frac{WX}{YX} = \frac{YX}{ZX}$

$\frac{5.5}{8.75} = \frac{8.75}{ZX}$ — $WX = 5.5$ and $YX = 8.75$

$5.5ZX = 76.5625$ — Cross products

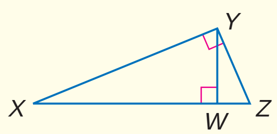
$ZX \approx 13.9$ — Divide each side by 5.5.

Mr. Martinez estimates that the elevated train is 5.5 + 13.9 or about 19.4 feet high.

The altitude to the hypotenuse of a right triangle determines another relationship between the segments.

Theorem 7.3

If the altitude is drawn from the vertex of the right angle of a right triangle to its hypotenuse, then the measure of a leg of the triangle is the geometric mean between the measures of the hypotenuse and the segment of the hypotenuse adjacent to that leg.



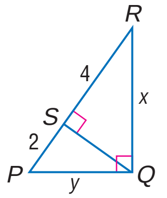
Example: $\frac{XZ}{XY} = \frac{XY}{XW}$ and $\frac{XZ}{YZ} = \frac{ZY}{WZ}$

You will prove Theorem 7.3 in Exercise 47.

Example 4 Hypotenuse and Segment of Hypotenuse

Find x and y in $\triangle PQR$.

$\overline{PQ}$ and $\overline{RQ}$ are legs of right triangle PQR. Use Theorem 7.3 to write a proportion for each leg and then solve.



$\frac{PR}{PQ} = \frac{PQ}{PS}$

$\frac{6}{y} = \frac{y}{2}$ — $PS = 2, PQ = y, PR = 6$

$y^2 = 12$ — Cross products

$y = \sqrt{12}$ — Take the square root.

$y = 2\sqrt{3}$ — Simplify.

$y \approx 3.5$ — Use a calculator.

$\frac{PR}{RQ} = \frac{RQ}{SR}$

$\frac{6}{x} = \frac{x}{4}$ — $RS = 4, RQ = x, PR = 6$

$x^2 = 24$ — Cross products

$x = \sqrt{24}$ — Take the square root.

$x = 2\sqrt{6}$ — Simplify.

$x \approx 4.9$ — Use a calculator.

Study Tip

Simplifying Radicals

Your teacher may ask you to simplify radicals, such as $\sqrt{12}$. Remember that $\sqrt{12} = \sqrt{4} \cdot \sqrt{3}$. Since $\sqrt{4} = 2$, $\sqrt{12} = 2\sqrt{3}$. For more practice simplifying radicals, see pages 744 and 745.

Check for Understanding

Concept Check

1. **OPEN ENDED** Find two numbers whose geometric mean is 12.

2. **Draw and label** a right triangle with an altitude drawn from the right angle. From your drawing, explain the meaning of *the hypotenuse and the segment of the hypotenuse adjacent to that leg* in Theorem 7.3.

3. **FIND THE ERROR** $\triangle RST$ is a right isosceles triangle. Holly and Ian are finding the measure of altitude $\overline{SU}$.

Holly

$\frac{RS}{SU} = \frac{SU}{RT}$

$\frac{9.9}{x} = \frac{x}{14}$

$x^2 = 138.6$

$x = \sqrt{138.6}$

$x \approx 11.8$

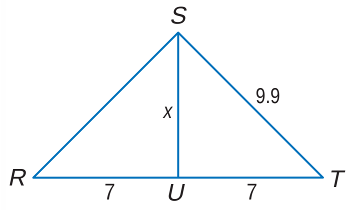
Ian

$\frac{RU}{SU} = \frac{SU}{UT}$

$\frac{7}{x} = \frac{x}{7}$

$x^2 = 49$

$x = 7$



Who is correct? Explain your reasoning.

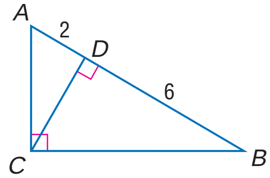
Guided Practice

Find the geometric mean between each pair of numbers.

4. 9 and 4
5. 36 and 49
6. 6 and 8
7. $2\sqrt{2}$ and $3\sqrt{2}$

Find the measure of each altitude.

8.



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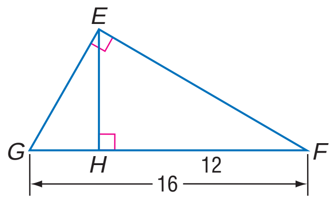
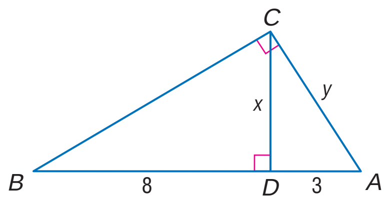


Find x and y.

10.



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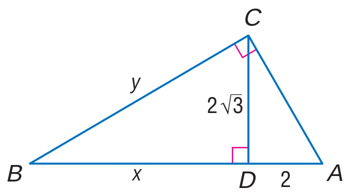
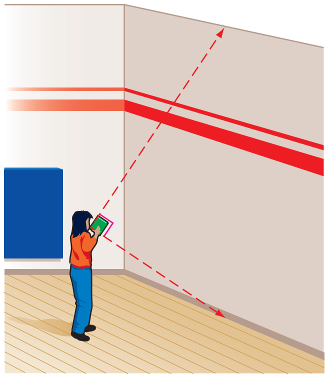


Application

12. DANCES Khaliah is making a banner for the dance committee. The banner is to be as high as the wall of the gymnasium. To find the height of the wall, Khaliah held a book up to her eyes so that the top and bottom of the wall were in line with the top edge and binding of the cover. If Khaliah's eye level is 5 feet off the ground and she is standing 12 feet from the wall, how high is the wall?



Practice and Apply

Homework Help	
For Exercises	See Examples
13–20	1
21–26	2
27–32	3, 4

Extra Practice
See page 766.

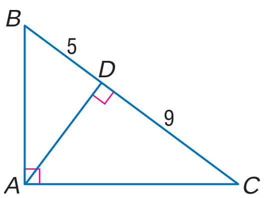
Find the geometric mean between each pair of numbers.

13. 5 and 6 **14.** 24 and 25 **15.** $\sqrt{45}$ and $\sqrt{80}$ **16.** $\sqrt{28}$ and $\sqrt{1372}$

17. $\frac{3}{5}$ and 1 **18.** $\frac{8\sqrt{3}}{5}$ and $\frac{6\sqrt{3}}{5}$ **19.** $\frac{2\sqrt{2}}{6}$ and $\frac{5\sqrt{2}}{6}$ **20.** $\frac{13}{7}$ and $\frac{5}{7}$

Find the measure of each altitude.

21.



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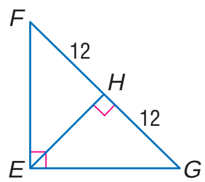

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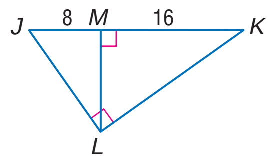

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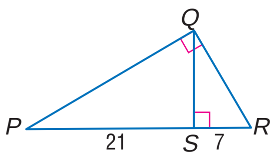

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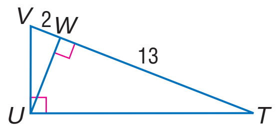

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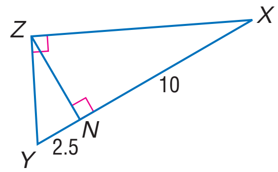
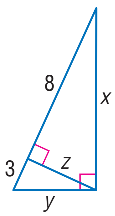


Find x, y, and z.

27.



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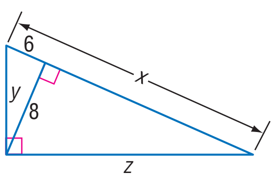

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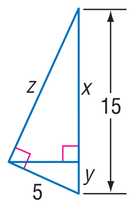

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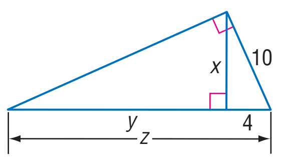

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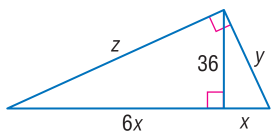

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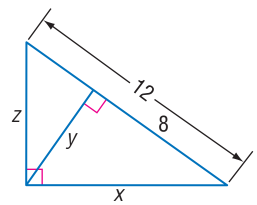


The geometric mean and one extreme are given. Find the other extreme.

33. $\sqrt{17}$ is the geometric mean between a and b. Find b if $a = 7$.

34. $\sqrt{12}$ is the geometric mean between x and y. Find x if $y = \sqrt{3}$.

Determine whether each statement is *always*, *sometimes*, or *never* true.

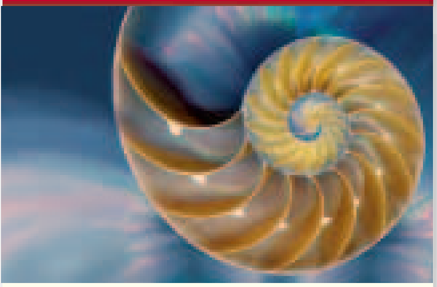
35. The geometric mean for consecutive positive integers is the average of the two numbers.

36. The geometric mean for two perfect squares is a positive integer.

37. The geometric mean for two positive integers is another integer.

38. The measure of the altitude of a triangle is the geometric mean between the measures of the segments of the side opposite the initial vertex.

More About. . .

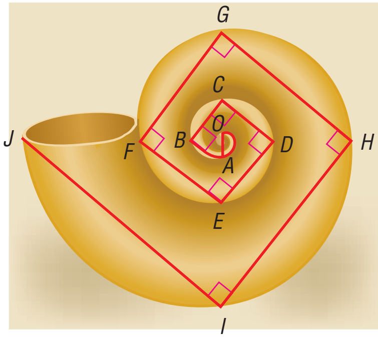


Chambered Nautilus

One of the most common traits of the chambered nautilus is that it is a natural example of the golden ratio.

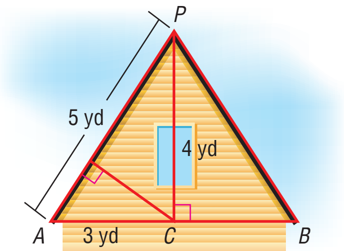
Source: www.infoplease.com

39. BIOLOGY The shape of the shell of a chambered nautilus can be modeled by a geometric mean. Consider the sequence of segments $\overline{OA}$, $\overline{OB}$, $\overline{OC}$, $\overline{OD}$, $\overline{OE}$, $\overline{OF}$, $\overline{OG}$, $\overline{OH}$, $\overline{OI}$, and $\overline{OJ}$. The length of each of these segments is the geometric mean between the lengths of the preceding segment and the succeeding segment. Explain this relationship. (*Hint:* Consider $\triangle FGH$.)



40. RESEARCH Refer to the information at the left. Use the Internet or other resource to write a brief description of the golden ratio.

41. CONSTRUCTION In the United States, most building codes limit the steepness of the slope of a roof to $\frac{4}{3}$, as shown at the right. A builder wants to put a support brace from point C perpendicular to $\overline{AP}$. Find the length of the brace.



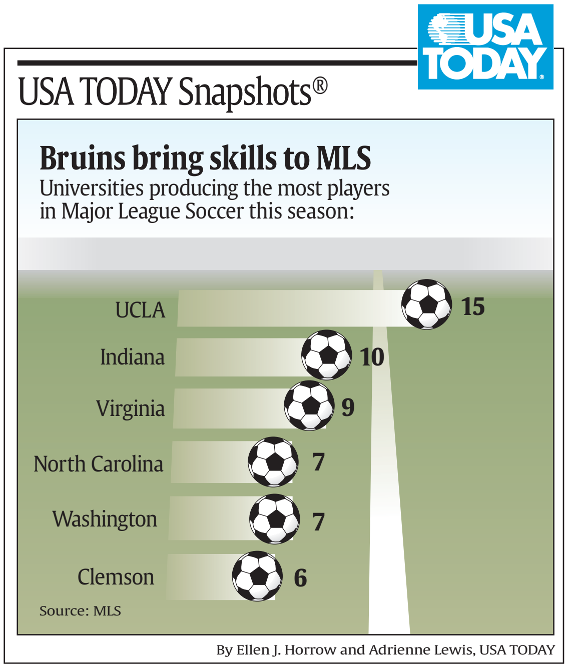
WebQuest

You can use geometric mean and the Quadratic Formula to discover the golden mean. Visit www.geometryonline.com/webquest to continue work on your WebQuest project.

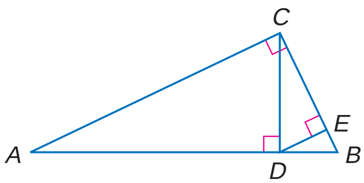
SOCCER For Exercises 42 and 43, refer to the graphic.

42. Find the geometric mean between the number of players from Indiana and North Carolina.

43. Are there two schools whose geometric mean is the same as the geometric mean between UCLA and Clemson? If so, which schools?



44. CRITICAL THINKING Find the exact value of DE, given $AD = 12$ and $BD = 4$.



www.geometryonline.com/self_check_quiz

Kaz Chiba/PhotoDisc

PROOF **Write the specified type proof for each theorem.**

45. two-column proof of Theorem 7.1

46. paragraph proof of Theorem 7.2

47. two-column proof of Theorem 7.3

48. *WRITING IN MATH* Answer the question that was posed at the beginning of the lesson.

How can the geometric mean be used to view paintings?

Include the following in your answer:

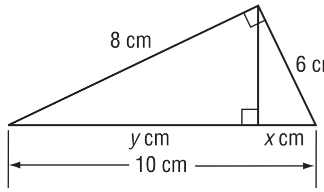
- an explanation of what happens when you are too far or too close to a painting, and
- an explanation of how the curator of a museum would determine where to place roping in front of paintings on display.

Standardized Test Practice

49. Find x and y.

(A) 4 and 6 (B) 2.5 and 7.5

(C) 3.6 and 6.4 (D) 3 and 7



50. **ALGEBRA** Solve $5x^2 + 405 = 1125$.

(A) 15 (B) 12

(C) $4\sqrt{3}$ (D) 4

Maintain Your Skills

Mixed Review

Find the first three iterations of each expression. *(Lesson 6-6)*

51. $x + 3$, where x initially equals 12

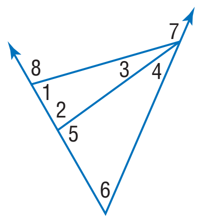
52. $3x + 2$, where x initially equals 4

53. $x^2 - 2$, where x initially equals 3

54. $2(x - 3)$, where x initially equals 1

55. The measures of the sides of a triangle are 20, 24, and 30. Find the measures of the segments formed where the bisector of the smallest angle meets the opposite side. *(Lesson 6-5)*

Use the Exterior Angle Inequality Theorem to list all angles that satisfy the stated condition. *(Lesson 5-2)*



56. all angles with a measure less than $m\angle 8$

57. all angles with a measure greater than $m\angle 1$

58. all angles with a measure less than $m\angle 7$

59. all angles with a measure greater than $m\angle 6$

Write an equation in slope-intercept form for the line that satisfies the given conditions. *(Lesson 3-4)*

60. $m = 2$, y-intercept $= 4$

61. x-intercept is 2, y-intercept $= -8$

62. passes through $(2, 6)$ and $(-1, 0)$

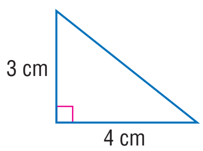
63. $m = -4$, passes through $(-2, -3)$

Getting Ready for the Next Lesson

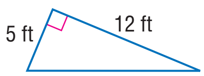
PREREQUISITE SKILL **Use the Pythagorean Theorem to find the length of the hypotenuse of each right triangle.**

*(To review **using the Pythagorean Theorem**, see Lesson 1-4.)*

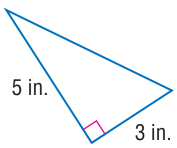
64.



65.



66.



Geometry Activity

A Preview of Lesson 7-2

The Pythagorean Theorem

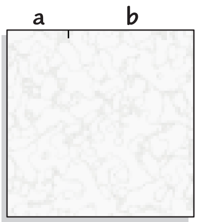
In Chapter 1, you learned that the Pythagorean Theorem relates the measures of the legs and the hypotenuse of a right triangle. Ancient cultures used the Pythagorean Theorem before it was officially named in 1909.

Use square pieces of patty paper and algebra. Then you too can discover this relationship among the measures of the sides of a right triangle.

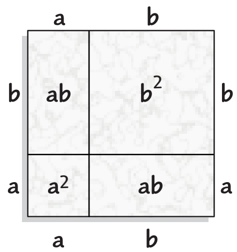
Activity

Use paper folding to develop the Pythagorean Theorem.

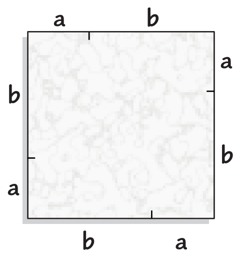
Step 1 On a piece of patty paper, make a mark along one side so that the two resulting segments are not congruent. Label one as a and the other as b.



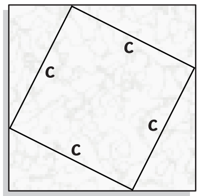
Step 2 Copy these measures on the other sides in the order shown at the right. Fold the paper to divide the square into four sections. Label the area of each section.



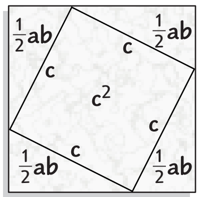
Step 3 On another sheet of patty paper, mark the same lengths a and b on the sides in the different pattern shown at the right.



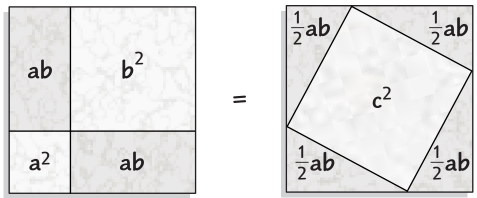
Step 4 Use your straightedge and pencil to connect the marks as shown at the right. Let c represent the length of each hypotenuse.



Step 5 Label the area of each section, which is $\frac{1}{2}ab$ for each triangle and c^2 for the square.



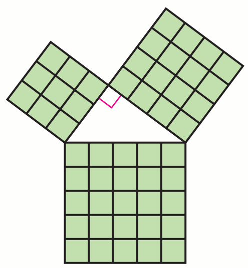
Step 6 Place the squares side by side and color the corresponding regions that have the same area. For example, $ab = \frac{1}{2}ab + \frac{1}{2}ab$.



The parts that are not shaded tell us that $a^2 + b^2 = c^2$.

Model

1. Use a ruler to find actual measures for a, b, and c. Do these measures measures confirm that $a^2 + b^2 = c^2$?
2. Repeat the activity with different a and b values. What do you notice?



Analyze the model

3. **Explain** why the drawing at the right is an illustration of the Pythagorean Theorem.

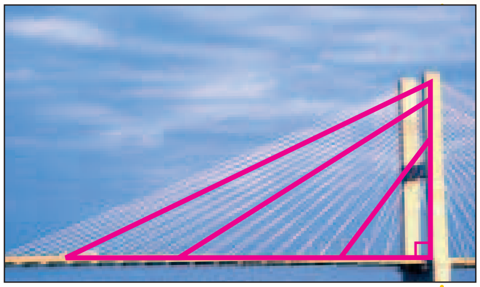
7-2 The Pythagorean Theorem and Its Converse

What You'll Learn

- Use the Pythagorean Theorem.
- Use the converse of the Pythagorean Theorem.

How are right triangles used to build suspension bridges?

The Talmadge Memorial Bridge over the Savannah River has two soaring towers of suspension cables. Note the right triangles being formed by the roadway, the perpendicular tower, and the suspension cables. The Pythagorean Theorem can be used to find measures in any right triangle.



Vocabulary

- Pythagorean triple

Study Tip

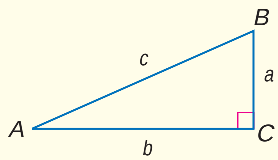
Look Back
To review **finding the hypotenuse of a right triangle,** see Lesson 1-3.

THE PYTHAGOREAN THEOREM

In Lesson 1-3, you used the Pythagorean Theorem to find the distance between two points by finding the length of the hypotenuse when given the lengths of the two legs of a right triangle. You can also find the measure of any side of a right triangle given the other two measures.

Theorem 7.4

Pythagorean Theorem In a right triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.



Symbols: $a^2 + b^2 = c^2$

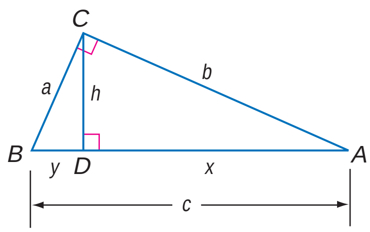
The geometric mean can be used to prove the Pythagorean Theorem.

Proof Pythagorean Theorem

Given: $\triangle ABC$ with right angle at C

Prove: $a^2 + b^2 = c^2$

Proof:



Draw right triangle ABC so C is the right angle. Then draw the altitude from C to $\overline{AB}$. Let $AB = c$, $AC = b$, $BC = a$, $AD = x$, $DB = y$, and $CD = h$.

Two geometric means now exist.

$$\frac{c}{a} = \frac{a}{y} \quad \text{and} \quad \frac{c}{b} = \frac{b}{x}$$

$$a^2 = cy \quad \text{and} \quad b^2 = cx \qquad \text{Cross products}$$

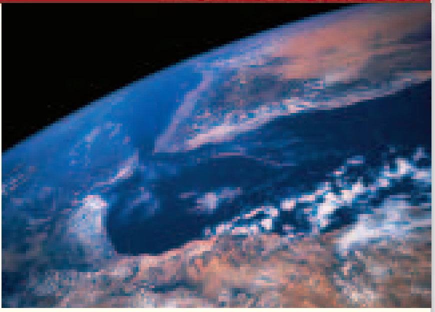
Add the equations.

$$a^2 + b^2 = cy + cx$$

$$a^2 + b^2 = c(y + x) \qquad \text{Factor.}$$

$$a^2 + b^2 = c^2 \qquad \text{Since } c = y + x \text{, substitute } c \text{ for } (y + x).$$

More About. . .



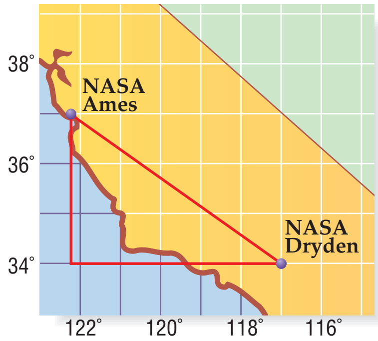
Maps

Due to the curvature of Earth, the distance between two points is often expressed as *degree distance* using latitude and longitude. This measurement closely approximates the distance on a plane.

Source: NASA

Example 1 Find the Length of the Hypotenuse

LONGITUDE AND LATITUDE **NASA Dryden is located at about 117 degrees longitude and 34 degrees latitude. NASA Ames is located at about 122 degrees longitude and 37 degrees latitude. Use the lines of longitude and latitude to find the degree distance to the nearest tenth between NASA Dryden and NASA Ames.**



The change in longitude between the two locations is $|117–122|$ or 5 degrees. Let this distance be a.

The change in latitude is $|37–34|$ or 3 degrees latitude. Let this distance be b.

Use the Pythagorean Theorem to find the distance in degrees from NASA Dryden to NASA Ames, represented by c.

$a^2 + b^2 = c^2$ Pythagorean Theorem

$5^2 + 3^2 = c^2$ $a = 5, b = 3$

$25 + 9 = c^2$ Simplify.

$34 = c^2$ Add.

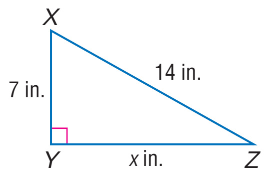
$\sqrt{34} = c$ Take the square root of each side.

$5.8 \approx c$ Use a calculator.

The degree distance between NASA Dryden and NASA Ames is about 5.8 degrees.

Example 2 Find the Length of a Leg

Find x.



$(XY)^2 + (YZ)^2 = (XZ)^2$ Pythagorean Theorem

$7^2 + x^2 = 14^2$ $XY = 7, XZ = 14$

$49 + x^2 = 196$ Simplify.

$x^2 = 147$ Subtract 49 from each side.

$x = \sqrt{147}$ Take the square root of each side.

$x = 7\sqrt{3}$ Simplify.

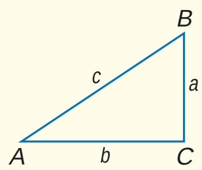
$x \approx 12.1$ Use a calculator.

CONVERSE OF THE PYTHAGOREAN THEOREM The converse of the Pythagorean Theorem can help you determine whether three measures of the sides of a triangle are those of a right triangle.

Theorem 7.5

Converse of the Pythagorean Theorem If the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

Symbols: If $a^2 + b^2 = c^2$, then $\triangle ABC$ is a right triangle.



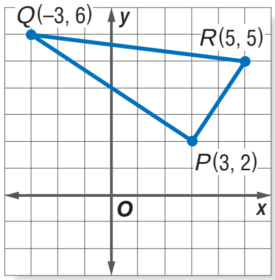
You will prove this theorem in Exercise 38.

Example 3 Verify a Triangle is a Right Triangle

Study Tip

Distance Formula
When using the Distance Formula, be sure to follow the order of operations carefully. Perform the operation inside the parentheses first, square each term, and then add.

COORDINATE GEOMETRY **Verify that $\triangle PQR$ is a right triangle.**



Use the Distance Formula to determine the lengths of the sides.

$$\begin{aligned} PQ &= \sqrt{(-3-3)^2 + (6-2)^2} && x_1 = 3, y_1 = 2, x_2 = -3, y_2 = 6 \\ &= \sqrt{(-6)^2 + 4^2} && \text{Subtract.} \\ &= \sqrt{52} && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} QR &= \sqrt{[5-(-3)]^2 + (5-6)^2} && x_1 = -3, y_1 = 6, x_2 = 5, y_2 = 5 \\ &= \sqrt{8^2 + (-1)^2} && \text{Subtract.} \\ &= \sqrt{65} && \text{Simplify.} \end{aligned}$$

$$\begin{aligned} PR &= \sqrt{(5-3)^2 + (5-2)^2} && x_1 = 3, y_1 = 2, x_2 = 5, y_2 = 5 \\ &= \sqrt{2^2 + 3^2} && \text{Subtract.} \\ &= \sqrt{13} && \text{Simplify.} \end{aligned}$$

By the converse of the Pythagorean Theorem, if the sum of the squares of the measures of two sides of a triangle equals the square of the measure of the longest side, then the triangle is a right triangle.

$$\begin{aligned} PQ^2 + PR^2 &= QR^2 && \text{Converse of the Pythagorean Theorem} \\ \left(\sqrt{52}\right)^2 + \left(\sqrt{13}\right)^2 &\stackrel{?}{=} \left(\sqrt{65}\right)^2 && PQ = \sqrt{52}, PR = \sqrt{13}, QR = \sqrt{65} \\ 52 + 13 &\stackrel{?}{=} 65 && \text{Simplify.} \\ 65 &= 65 && \text{Add.} \end{aligned}$$

Since the sum of the squares of two sides equals the square of the longest side, $\triangle PQR$ is a right triangle.

A **Pythagorean triple** is three whole numbers that satisfy the equation $a^2 + b^2 = c^2$, where c is the greatest number. One common Pythagorean triple is 3-4-5, in which the sides of a right triangle are in the ratio 3:4:5. If the measures of the sides of any right triangle are whole numbers, the measures form a Pythagorean triple.

Example 4 Pythagorean Triples

Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple.

a. 8, 15, 16

Since the measure of the longest side is 16, 16 must be c, and a or b are 15 and 8.

$$\begin{aligned} a^2 + b^2 &= c^2 && \text{Pythagorean Theorem} \\ 8^2 + 15^2 &\stackrel{?}{=} 16^2 && a = 8, b = 15, c = 16 \\ 64 + 225 &\stackrel{?}{=} 256 && \text{Simplify.} \\ 289 &\neq 256 && \text{Add.} \end{aligned}$$

Since $289 \neq 256$, segments with these measures cannot form a right triangle. Therefore, they do not form a Pythagorean triple.

CONTENTS

b. 20, 48, and 52

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$
$$20^2 + 48^2 \stackrel{?}{=} 52^2 \quad a = 20, b = 48, c = 52$$
$$400 + 2304 \stackrel{?}{=} 2704 \quad \text{Simplify.}$$
$$2704 = 2704 \quad \text{Add.}$$

These segments form the sides of a right triangle since they satisfy the Pythagorean Theorem. The measures are whole numbers and form a Pythagorean triple.

c. $\frac{\sqrt{3}}{5}$, $\frac{\sqrt{6}}{5}$, **and** $\frac{3}{5}$

$$a^2 + b^2 = c^2 \quad \text{Pythagorean Theorem}$$
$$\left(\frac{\sqrt{3}}{5}\right)^2 + \left(\frac{\sqrt{6}}{5}\right)^2 \stackrel{?}{=} \left(\frac{3}{5}\right)^2 \quad a = \frac{\sqrt{3}}{5}, b = \frac{\sqrt{6}}{5}, c = \frac{3}{5}$$
$$\frac{3}{25} + \frac{6}{25} \stackrel{?}{=} \frac{9}{25} \quad \text{Simplify.}$$
$$\frac{9}{25} = \frac{9}{25} \quad \text{Add.}$$

Since $\frac{9}{25} = \frac{9}{25}$, segments with these measures form a right triangle. However, the three numbers are not whole numbers. Therefore, they do not form a Pythagorean triple.

Study Tip

Comparing Numbers

If you cannot quickly identify the greatest number, use a calculator to find decimal values for each number and compare.

Check for Understanding

Concept Check

1. **FIND THE ERROR** Maria and Colin are determining whether 5-12-13 is a Pythagorean triple.

Colin	Maria
$a^2 + b^2 = c^2$	$a^2 + b^2 = c^2$
$13^2 + 5^2 \stackrel{?}{=} 12^2$	$5^2 + 12^2 \stackrel{?}{=} 13^2$
$169 + 25 \stackrel{?}{=} 144$	$25 + 144 \stackrel{?}{=} 169$
$193 \neq 144$	$169 = 169$
no	yes

Who is correct? Explain your reasoning.

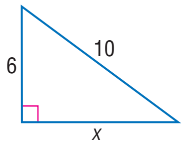
2. **Explain** why a Pythagorean triple can represent the measures of the sides of a right triangle.

3. **OPEN ENDED** Draw a pair of similar right triangles. List the corresponding sides, the corresponding angles, and the scale factor. Are the measures of the sides of each triangle a Pythagorean triple?

Guided Practice

Find *x*.

4.



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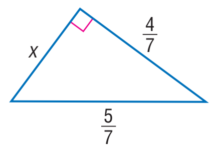

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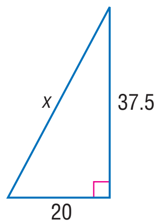


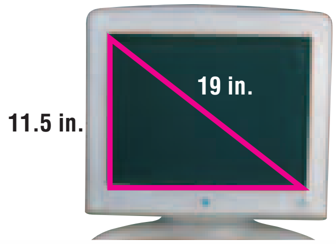
7. **COORDINATE GEOMETRY** Determine whether $\triangle JKL$ with vertices $J(-2, 2)$, $K(-1, 6)$, and $L(3, 5)$ is a right triangle. Explain.

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

8. 15, 36, 39 **9.** $\sqrt{40}$, 20, 21 **10.** $\sqrt{44}$, 8, $\sqrt{108}$

Application

11. **COMPUTERS** Computer monitors are usually measured along the diagonal of the screen. A 19-inch monitor has a diagonal that measures 19 inches. If the height of the screen is 11.5 inches, how wide is the screen?



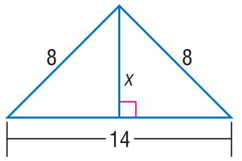
Practice and Apply

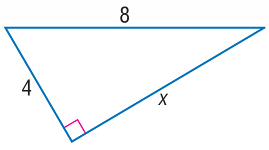
Homework Help

For Exercises	See Examples
14, 15	1
12, 13, 16, 17	2
18–21	3
22–29	4

Extra Practice
See page 767.

Find *x*.

12.


13.


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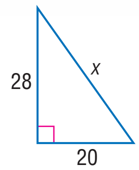
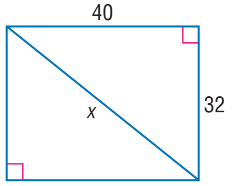
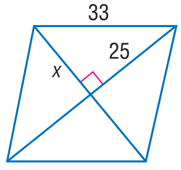
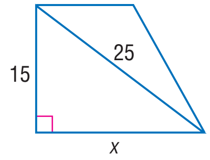


15.


16.


17.


COORDINATE GEOMETRY **Determine whether $\triangle QRS$ is a right triangle for the given vertices. Explain.**

18. $Q(1, 0)$, $R(1, 6)$, $S(9, 0)$ **19.** $Q(3, 2)$, $R(0, 6)$, $S(6, 6)$

20. $Q(-4, 6)$, $R(2, 11)$, $S(4, -1)$ **21.** $Q(-9, -2)$, $R(-4, -4)$, $S(-6, -9)$

Determine whether each set of numbers can be the measures of the sides of a right triangle. Then state whether they form a Pythagorean triple.

22. 8, 15, 17 **23.** 7, 24, 25 **24.** 20, 21, 31 **25.** 37, 12, 34

26. $\frac{1}{5}, \frac{1}{7}, \frac{\sqrt{74}}{35}$ **27.** $\frac{\sqrt{3}}{2}, \frac{\sqrt{2}}{3}, \frac{35}{36}$ **28.** $\frac{3}{5}, \frac{4}{5}, 1$ **29.** $\frac{6}{7}, \frac{8}{7}, \frac{10}{7}$

For Exercises 30–35, use the table of Pythagorean triples.

a	*b*	*c*
5	12	13
10	24	?
15	?	39
?	48	52

30. Copy and complete the table.

31. A *primitive* Pythagorean triple is a Pythagorean triple with no common factors except 1. Name any primitive Pythagorean triples contained in the table.

32. Describe the pattern that relates these sets of Pythagorean triples.

33. These Pythagorean triples are called a *family*. Why do you think this is?

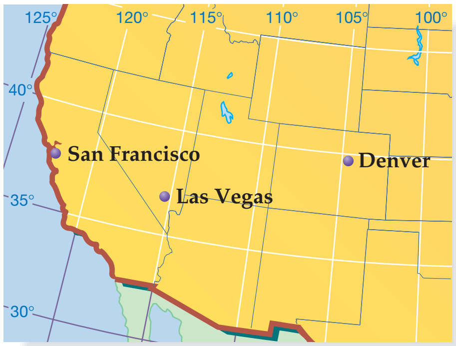
34. Are the triangles described by a family of Pythagorean triples similar? Explain.

35. For each Pythagorean triple, find two triples in the same family.

a. 8, 15, 17 **b.** 9, 40, 41 **c.** 7, 24, 25

GEOGRAPHY **For Exercises 36 and 37, use the following information.**

Denver is located at about 105 degrees longitude and 40 degrees latitude. San Francisco is located at about 122 degrees longitude and 38 degrees latitude. Las Vegas is located at about 115 degrees longitude and 36 degrees latitude. Using the lines of longitude and latitude, find each degree distance.

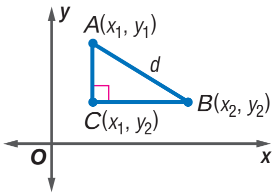


36. San Francisco to Denver

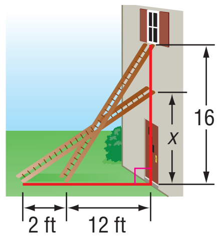
37. Las Vegas to Denver

38. **PROOF** Write a paragraph proof of Theorem 7.5.

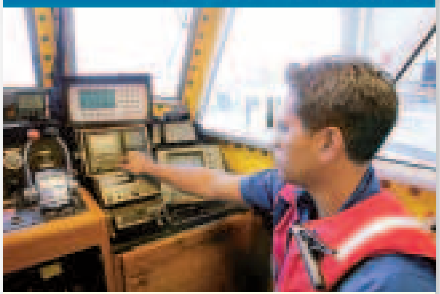
39. **PROOF** Use the Pythagorean Theorem and the figure at the right to prove the Distance Formula.



40. **PAINTING** A painter sets a ladder up to reach the bottom of a second-story window 16 feet above the ground. The base of the ladder is 12 feet from the house. While the painter mixes the paint, a neighbor's dog bumps the ladder, which moves the base 2 feet farther away from the house. How far up the side of the house does the ladder reach?



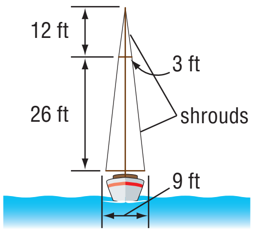
Career Choices



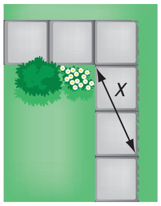
Military All branches of the military use navigation. Some of the jobs using navigation include radar/sonar operators, boat operators, airplane navigators, and space operations officers.

Online Research For information about a career in the military, visit: www.geometryonline.com/careers

41. **SAILING** The mast of a sailboat is supported by wires called *shrouds*. What is the total length of wire needed to form these shrouds?

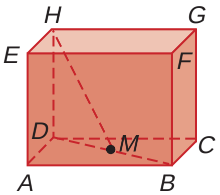


42. **LANDSCAPING** Six congruent square stones are arranged in an L-shaped walkway through a garden. If $x = 15$ inches, then find the area of the L-shaped walkway.



43. **NAVIGATION** A fishing trawler off the coast of Alaska was ordered by the U.S. Coast Guard to change course. They were to travel 6 miles west and then sail 12 miles south to miss a large iceberg before continuing on the original course. How many miles out of the way did the trawler travel?

44. **CRITICAL THINKING** The figure at the right is a rectangular prism with $AB = 8$, $BC = 6$, and $BF = 8$, and M is the midpoint of $\overline{BD}$. Find BD and HM. How are EM, FM, and GM related to HM?



45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are right triangles used to build suspension bridges?

Include the following in your answer:

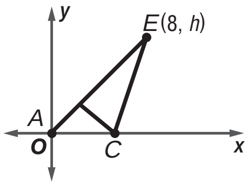
- the locations of the right triangles, and
- an explanation of which parts of the right triangle are formed by the cables.

Standardized Test Practice

46. In the figure, if $AE = 10$, what is the value of h?

Ⓐ 6 Ⓑ 8

Ⓒ 10 Ⓓ 12



47. **ALGEBRA** If $x^2 + 36 = (9 - x)^2$, then find x.

Ⓐ 6 Ⓑ no solution

Ⓒ 2.5 Ⓓ 10

Graphing Calculator

PROGRAMMING For Exercises 48 and 49, use the following information.

The TI-83 Plus program uses a procedure for finding *Pythagorean triples* that was developed by Euclid around 320 B.C. Run the program to generate a list of Pythagorean triples.

```
PROGRAM: PYTHTRIP
:For (X, 2, 6)
:For (Y, 1, 5)
:If X > Y
:Then
:int (X²–Y² + 0.5)→A
:2XY→B
:int (X²+Y²+0.5)→C
:If A > B
:Then
:Disp B,A,C
:Else
:Disp A,B,C
:End
:End
:End
:Pause
:Disp " "
:End
:End
:Stop
```

48. List all the members of the 3-4-5 family that are generated by the program.

49. A geometry student made the conjecture that if three whole numbers are a Pythagorean triple, then their product is divisible by 60. Does this conjecture hold true for each triple that is produced by the program?

Maintain Your Skills

Mixed Review

Find the geometric mean between each pair of numbers. *(Lesson 7-1)*

50. 3 and 12 51. 9 and 12 52. 11 and 7

53. 6 and 9 54. 2 and 7 55. 2 and 5

Find the value of each expression. Then use that value as the next x in the expression. Repeat the process and describe your observations. *(Lesson 6-6)*

56. $\sqrt{2x}$, where x initially equals 5 57. 3^x, where x initially equals 1

58. $x^{\frac{1}{2}}$, where x initially equals 4 59. $\frac{1}{x}$, where x initially equals 4

60. Determine whether the sides of a triangle could have the lengths 12, 13, and 25. Explain. *(Lesson 5-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Simplify each expression by rationalizing the denominator.**

(To review ***simplifying radical expressions****, see pages 744 and 745.)*

61. $\frac{7}{\sqrt{3}}$ 62. $\frac{18}{\sqrt{2}}$ 63. $\frac{\sqrt{14}}{\sqrt{2}}$ 64. $\frac{3\sqrt{11}}{\sqrt{3}}$ 65. $\frac{24}{\sqrt{2}}$

66. $\frac{12}{\sqrt{3}}$ 67. $\frac{2\sqrt{6}}{\sqrt{3}}$ 68. $\frac{15}{\sqrt{3}}$ 69. $\frac{2}{\sqrt{8}}$ 70. $\frac{25}{\sqrt{10}}$

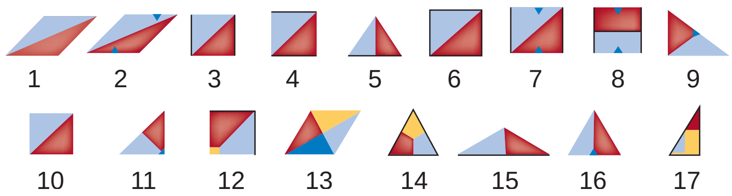
7-3 Special Right Triangles

What You'll Learn

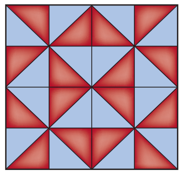
- Use properties of 45°-45°-90° triangles.
- Use properties of 30°-60°-90° triangles.

How is triangle tiling used in wallpaper design?

Triangle tiling is the process of laying copies of a single triangle next to each other to fill an area. One type of triangle tiling is *wallpaper tiling*. There are exactly 17 types of triangle tiles that can be used for wallpaper tiling.

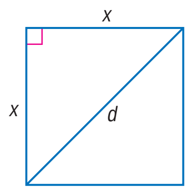


Tile 4 is made up of two 45°-45°-90° triangles that form a square. This tile is rotated to make the wallpaper design shown at the right



PROPERTIES OF 45°-45°-90° TRIANGLES Facts about 45°-45°-90° triangles are used to solve many geometry problems. The Pythagorean Theorem allows us to discover special relationships that exist among the sides of a 45°-45°-90° triangle.

Draw a diagonal of a square. The two triangles formed are isosceles right triangles. Let x represent the measure of each side and let d represent the measure of the hypotenuse.

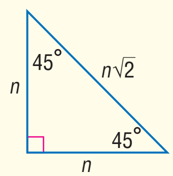


$d^2 = x^2 + x^2$	Pythagorean Theorem
$d^2 = 2x^2$	Add.
$d = \sqrt{2x^2}$	Take the positive square root of each side.
$d = \sqrt{2} \cdot \sqrt{x^2}$	Factor.
$d = x\sqrt{2}$	Simplify.

This algebraic proof verifies that the length of the hypotenuse of any 45°-45°-90° triangle is $\sqrt{2}$ times the length of its leg. The ratio of the sides is $1 : 1 : \sqrt{2}$.

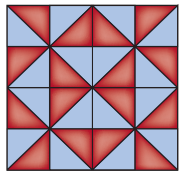
Theorem 7.6

In a 45°-45°-90° triangle, the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.



Example 1 Find the Measure of the Hypotenuse

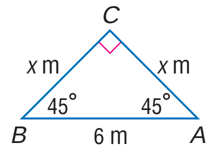
WALLPAPER TILING **Assume that the length of one of the legs of the 45°-45°-90° triangles in the wallpaper in the figure is 4 inches. What is the length of the diagonal of the entire wallpaper square?**



The length of each leg of the 45°-45°-90° triangle is 4 inches. The length of the hypotenuse is $\sqrt{2}$ times as long as a leg. The length of the hypotenuse of one of the triangles is $4\sqrt{2}$. There are four 45°-45°-90° triangles along the diagonal of the square. So, the length of the diagonal of the square is $4(4\sqrt{2})$ or $16\sqrt{2}$ inches.

Example 2 Find the Measure of the Legs

Find x.



The length of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg of the triangle.

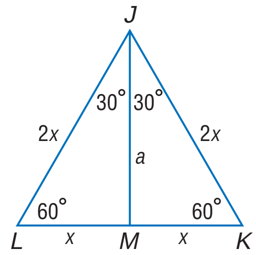
$AB = (AC)\sqrt{2}$	
$6 = x\sqrt{2}$	$AB = 6$, $AC = x$
$\frac{6}{\sqrt{2}} = x$	Divide each side by $\sqrt{2}$.
$\frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = x$	Rationalize the denominator.
$\frac{6\sqrt{2}}{2} = x$	Multiply.
$3\sqrt{2} = x$	Divide.

Study Tip

Rationalizing Denominators

To rationalize a denominator, multiply the fraction by 1 in the form of a radical over itself so that the product in the denominator is a rational number.

PROPERTIES OF 30°-60°-90° TRIANGLES There is also a special relationship among the measures of the sides of a 30°-60°-90° triangle.



When an altitude is drawn from any vertex of an equilateral triangle, two congruent 30°-60°-90° triangles are formed. $\overline{LM}$ and $\overline{KM}$ are congruent segments, so let $LM = x$ and $KM = x$. By the Segment Addition Postulate, $LM + KM = KL$. Thus, $KL = 2x$. Since $\triangle JKL$ is an equilateral triangle, $KL = JL = JK$. Therefore, $JL = 2x$ and $JK = 2x$.

Let a represent the measure of the altitude. Use the Pythagorean Theorem to find a.

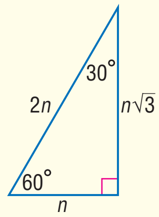
$(JM)^2 + (LM)^2 = (JL)^2$	Pythagorean Theorem
$a^2 + x^2 = (2x)^2$	$JM = a$, $LM = x$, $JL = 2x$
$a^2 + x^2 = 4x^2$	Simplify.
$a^2 = 3x^2$	Subtract x^2 from each side.
$a = \sqrt{3x^2}$	Take the positive square root of each side.
$a = \sqrt{3} \cdot \sqrt{x^2}$	Factor.
$a = x\sqrt{3}$	Simplify.

So, in a 30°-60°-90° triangle, the measures of the sides are x, $x\sqrt{3}$, and $2x$. The ratio of the sides is $1 : \sqrt{3} : 2$.

The relationship of the side measures leads to Theorem 7.7.

Theorem 7.7

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg, and the length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.



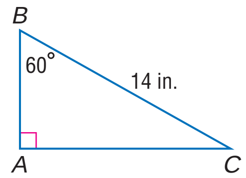
Study Tip

30°-60°-90° Triangle
The shorter leg is opposite the 30° angle, and the longer leg is opposite the 60° angle.

Example 3 30°-60°-90° Triangles

Find *AC*.

$\overline{AC}$ is the longer leg, $\overline{AB}$ is the shorter leg, and $\overline{BC}$ is the hypotenuse.



$$AB = \frac{1}{2}(BC)$$

$$= \frac{1}{2}(14) \text{ or } 7 \qquad BC = 14$$

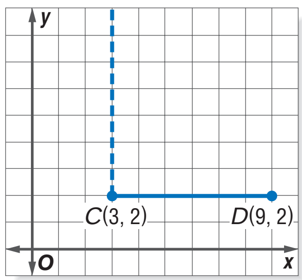
$$AC = \sqrt{3}(AB)$$

$$= \sqrt{3}(7) \text{ or } 7\sqrt{3} \qquad AB = 7$$

Example 4 Special Triangles in a Coordinate Plane

COORDINATE GEOMETRY **Triangle *PCD* is a 30°-60°-90° triangle with right angle *C*. $\overline{CD}$ is the longer leg with endpoints *C*(3, 2) and *D*(9, 2). Locate point *P* in Quadrant I.**

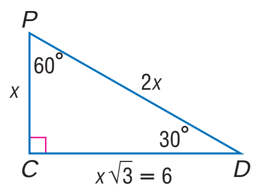
Graph *C* and *D*. $\overline{CD}$ lies on a horizontal gridline of the coordinate plane. Since $\overline{PC}$ will be perpendicular to $\overline{CD}$, it lies on a vertical gridline. Find the length of $\overline{CD}$.



$$CD = |9 - 3| = 6$$

$\overline{CD}$ is the longer leg. $\overline{PC}$ is the shorter leg.

So, $CD = \sqrt{3}(PC)$. Use *CD* to find *PC*.



$$CD = \sqrt{3}(PC)$$

$$6 = \sqrt{3}(PC) \qquad CD = 6$$

$$\frac{6}{\sqrt{3}} = PC \qquad \text{Divide each side by } \sqrt{3}.$$

$$\frac{6}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = PC \qquad \text{Rationalize the denominator.}$$

$$\frac{6\sqrt{3}}{3} = PC \qquad \text{Multiply.}$$

$$2\sqrt{3} = PC \qquad \text{Simplify.}$$

Point *P* has the same *x*-coordinate as *C*. *P* is located $2\sqrt{3}$ units above *C*. So, the coordinates of *P* are $(3, 2 + 2\sqrt{3})$ or about (3, 5.46).

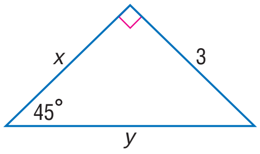
Check for Understanding

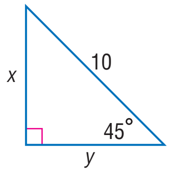
Concept Check

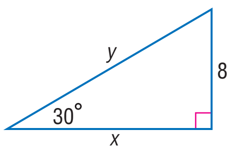
1. **OPEN ENDED** Draw a 45°-45°-90° triangle. Be sure to label the angles and the sides and to explain how you made the drawing.
2. **Draw** a 30°-60°-90° triangle with the shorter leg 2 centimeters long. Label the angles and the remaining sides.
3. **Write** an equation to find the length of a rectangle that has a diagonal twice as long as its width.

Guided Practice

Find *x* and *y*.

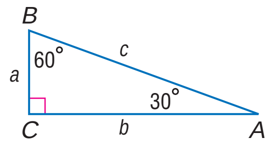
4. 

5. 

6. 

Find the missing measures.

7. If $c = 8$, find a and b.
8. If $b = 18$, find a and c.

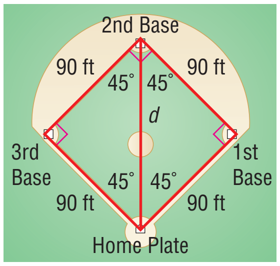


Triangle *ABD* is a 30°-60°-90° triangle with right angle *B* and with $\overline{AB}$ as the shorter leg. Graph *A* and *B*, and locate point *D* in Quadrant I.

9. $A(8, 0)$, $B(8, 3)$
10. $A(6, 6)$, $B(2, 6)$

Application

11. **SOFTBALL** Find the distance from home plate to second base if the bases are 90 feet apart.



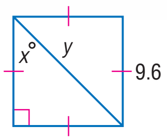
Practice and Apply

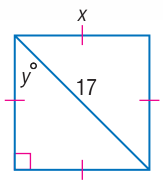
Homework Help

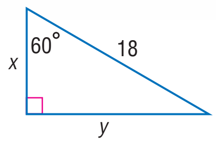
For Exercises	See Examples
12, 13, 17, 22, 25	1, 2
14–16, 18–21, 23, 24	3
27–31	4

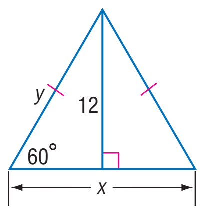
Extra Practice
See page 767.

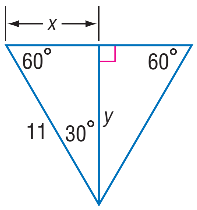
Find *x* and *y*.

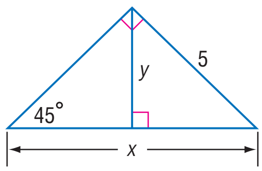
12. 

13. 

14. 

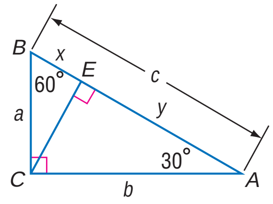
15. 

16. 

17. 

For Exercises 18 and 19, use the figure at the right.

18. If $a = 10\sqrt{3}$, find CE and y.
19. If $x = 7\sqrt{3}$, find a, CE, y, and b.



20. The length of an altitude of an equilateral triangle is 12 feet. Find the length of a side of the triangle.

21. The perimeter of an equilateral triangle is 45 centimeters. Find the length of an altitude of the triangle.

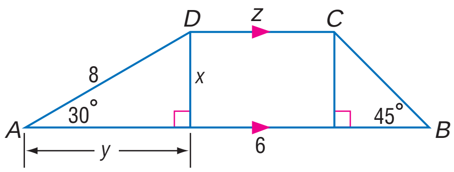
22. The length of a diagonal of a square is $22\sqrt{2}$ millimeters. Find the perimeter of the square.

23. The altitude of an equilateral triangle is 7.4 meters long. Find the perimeter of the triangle.

24. The diagonals of a rectangle are 12 inches long and intersect at an angle of 60°. Find the perimeter of the rectangle.

25. The sum of the squares of the measures of the sides of a square is 256. Find the measure of a diagonal of the square.

26. Find x, y, z, and the perimeter of $ABCD$.



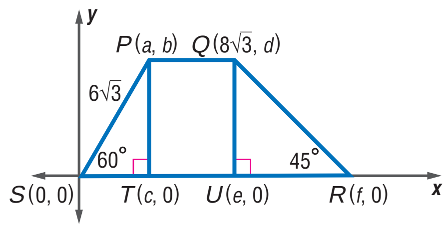
27. $\triangle PAB$ is a 45°-45°-90° triangle with right angle B. Find the coordinates of P in Quadrant I for $A(-3, 1)$ and $B(4, 1)$.

28. $\triangle PGH$ is a 45°-45°-90° triangle with $m\angle P = 90$. Find the coordinates of P in Quadrant I for $G(4, -1)$ and $H(4, 5)$.

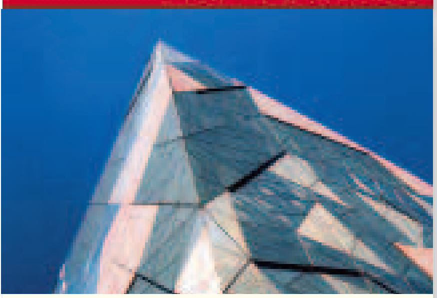
29. $\triangle PCD$ is a 30°-60°-90° triangle with right angle C and $\overline{CD}$ the longer leg. Find the coordinates of P in Quadrant III for $C(-3, -6)$ and $D(-3, 7)$.

30. $\triangle PCD$ is a 30°-60°-90° triangle with $m\angle C = 30$ and hypotenuse $\overline{CD}$. Find the coordinates of P for $C(2, -5)$ and $D(10, -5)$ if P lies above $\overline{CD}$.

31. If $\overline{PQ} \parallel \overline{SR}$, use the figure to find a, b, c, and d.



More About. . .

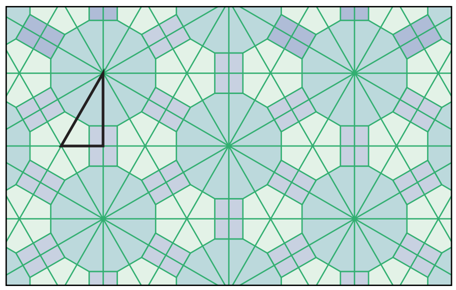


Triangle Tiling

Buildings in Federation Square in Melbourne, Australia, feature a tiling pattern called a pinwheel tiling. The sides of each right triangle are in the ratio $1:2:\sqrt{5}$.

Source: www.federationsquare.com.au

TRIANGLE TILING **For Exercises 32–35, use the following information.** *Triangle tiling* refers to the process of taking many copies of a single triangle and laying them next to each other to fill an area. For example, the pattern shown is composed of tiles like the one outlined.



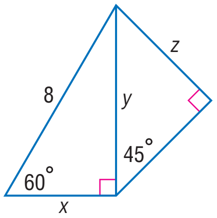
32. How many 30°-60°-90° triangles are used to create the basic circular pattern?

33. Which angle of the 30°-60°-90° triangle is being rotated to make the basic shape?

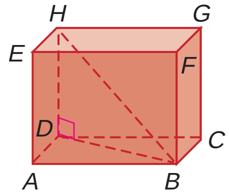
34. Explain why there are no gaps in the basic pattern.

35. Use grid paper to cut out 30°-60°-90° triangles. Color the same pattern on each triangle. Create one basic figure that would be part of a wallpaper tiling.

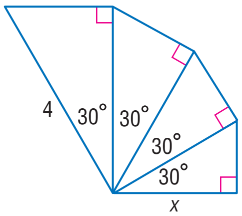
36. Find x, y, and z.



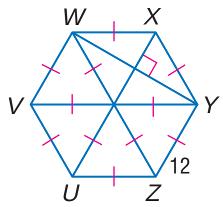
37. If $BD = 8\sqrt{3}$ and $m\angle DHB = 60$, find BH.



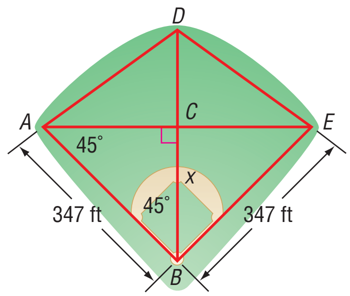
38. Each triangle in the figure is a 30°-60°-90° triangle. Find x.



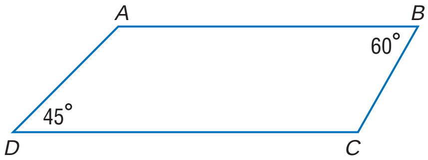
39. In regular hexagon $UVWXYZ$, each side is 12 centimeters long. Find WY.



40. **BASEBALL** The diagram at the right shows some dimensions of Comiskey Park in Chicago, Illinois. $\overline{BD}$ is a segment from home plate to dead center field, and $\overline{AE}$ is a segment from the left field foul-ball pole to the right field foul-ball pole. If the center fielder is standing at C, how far is he from home plate?



41. **CRITICAL THINKING** Given figure $ABCD$, with $\overline{AB} \parallel \overline{DC}$, $m\angle B = 60$, $m\angle D = 45$, $BC = 8$, and $AB = 24$, find the perimeter.



42. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How is triangle tiling used in wallpaper design?

Include the following in your answer:

- which of the numbered designs contain 30°-60°-90° triangles and which contain 45°-45°-90° triangles, and
- a reason why rotations of the basic design left no holes in the completed design.

Standardized Test Practice

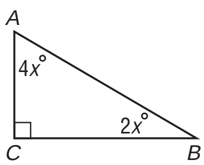
43. In the right triangle, what is AB if $BC = 6$?

Ⓐ 12 units

Ⓑ $6\sqrt{2}$ units

Ⓒ $4\sqrt{3}$ units

Ⓓ $2\sqrt{3}$ units



44. **SHORT RESPONSE** For real numbers a and b, where $b \neq 0$, if $a \star b = \frac{a^2}{b^2}$, then $(3 \star 4)(5 \star 3) =$ __?__.

Maintain Your Skills

Mixed Review

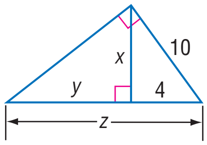
Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple. *(Lesson 7-2)*

45. 3, 4, 5 **46.** 9, 40, 41 **47.** 20, 21, 31

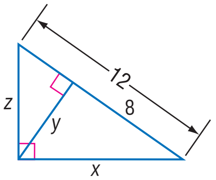
48. 20, 48, 52 **49.** 7, 24, 25 **50.** 12, 34, 37

Find x, y, and z. *(Lesson 7-1)*

51.



52.



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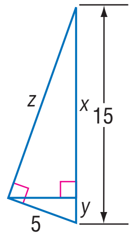
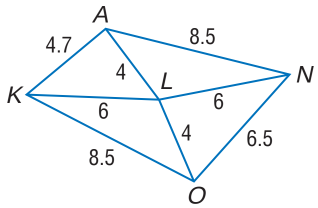


Write an inequality relating each pair of angles. *(Lesson 5-5)*



54. $m\angle ALK$, $m\angle ALN$

55. $m\angle ALK$, $m\angle NLO$

56. $m\angle OLK$, $m\angle NLO$

57. $m\angle KLO$, $m\angle ALN$

58. Determine whether $\triangle JKL$ with vertices $J(-3, 2)$, $K(-1, 5)$, and $L(4, 4)$ is congruent to $\triangle RST$ with vertices $R(-6, 6)$, $S(-4, 3)$, and $T(1, 4)$. Explain. *(Lesson 4-4)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each equation.**

*(To review **solving equations**, see pages 737 and 738.)*

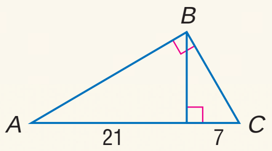
59. $5 = \frac{x}{3}$ **60.** $\frac{x}{9} = 0.14$ **61.** $0.5 = \frac{10}{k}$ **62.** $0.2 = \frac{13}{g}$

63. $\frac{7}{n} = 0.25$ **64.** $9 = \frac{m}{0.8}$ **65.** $\frac{24}{x} = 0.4$ **66.** $\frac{35}{y} = 0.07$

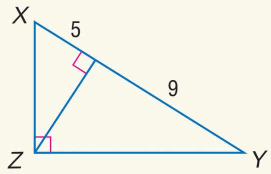
Practice Quiz 1 — *Lessons 7-1 through 7-3*

Find the measure of each altitude. *(Lesson 7-1)*

1.



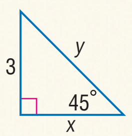
2.



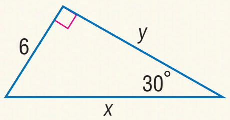
3. Determine whether $\triangle ABC$ with vertices $A(2, 1)$, $B(4, 0)$, and $C(5, 7)$ is a right triangle. Explain. *(Lesson 7-2)*

Find x and y. *(Lesson 7-3)*

4.



5.



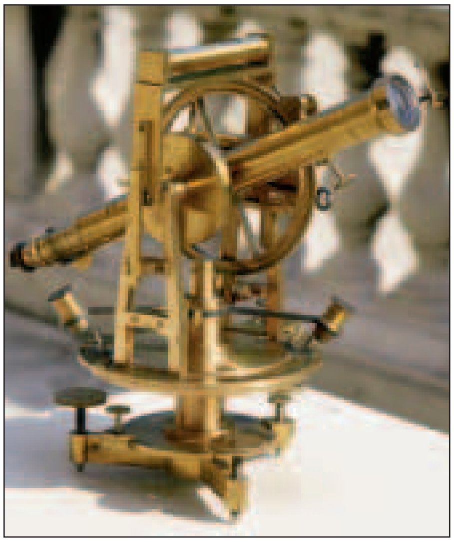
7-4 Trigonometry

What You'll Learn

- Find trigonometric ratios using right triangles.
- Solve problems using trigonometric ratios.

How can surveyors determine angle measures?

The old surveyor's telescope shown at right is called a theodolite (thee AH duh lite). It is an optical instrument used to measure angles in surveying, navigation, and meteorology. It consists of a telescope fitted with a level and mounted on a tripod so that it is free to rotate about its vertical and horizontal axes. After measuring angles, surveyors apply trigonometry to calculate distance or height.

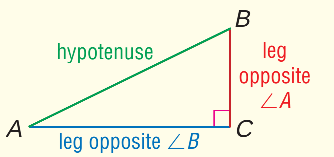
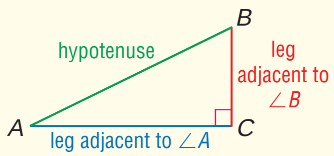
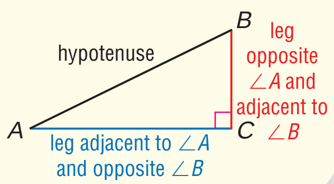


Vocabulary

- trigonometry
- trigonometric ratio
- sine
- cosine
- tangent

TRIGONOMETRIC RATIOS The word **trigonometry** comes from two Greek terms, *trigon*, meaning triangle, and *metron*, meaning measure. The study of trigonometry involves triangle measurement. A ratio of the lengths of sides of a right triangle is called a **trigonometric ratio**. The three most common trigonometric ratios are **sine**, **cosine**, and **tangent**.

Key Concept — *Trigonometric Ratios*

Words	Symbols	Models
sine of $\angle A = \dfrac{\text{measure of leg opposite } \angle A}{\text{measure of hypotenuse}}$	$\sin A = \dfrac{BC}{AB}$	
sine of $\angle B = \dfrac{\text{measure of leg opposite } \angle B}{\text{measure of hypotenuse}}$	$\sin B = \dfrac{AC}{AB}$	
cosine of $\angle A = \dfrac{\text{measure of leg adjacent to } \angle A}{\text{measure of hypotenuse}}$	$\cos A = \dfrac{AC}{AB}$	
cosine of $\angle B = \dfrac{\text{measure of leg adjacent to } \angle B}{\text{measure of hypotenuse}}$	$\cos B = \dfrac{BC}{AB}$	
tangent of $\angle A = \dfrac{\text{measure of leg opposite } \angle A}{\text{measure of leg adjacent to } \angle A}$	$\tan A = \dfrac{BC}{AC}$	
tangent of $\angle B = \dfrac{\text{measure of leg opposite } \angle B}{\text{measure of leg adjacent to } \angle B}$	$\tan B = \dfrac{AC}{BC}$	

Study Tip

Reading Math
SOH-CAH-TOA is a mnemonic device for learning the ratios for sine, cosine, and tangent using the first letter of each word in the ratios.

$\sin A = \dfrac{opp}{hyp}$

$\cos A = \dfrac{adj}{hyp}$

$\tan A = \dfrac{opp}{adj}$

Trigonometric ratios are related to the acute angles of a right triangle, *not* the right angle.

Study Tip

Equivalent Ratios

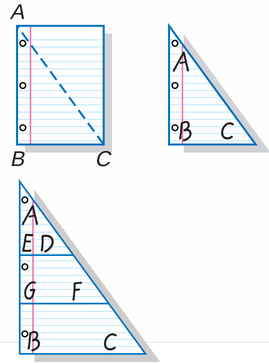
Notice that the ratio $\frac{\text{leg opposite } \angle A}{\text{hypotenuse}}$ is the same as $\frac{\text{leg adjacent } \angle C}{\text{hypotenuse}}$.

Thus, $\sin A = \cos C = \frac{a}{b}$.

Likewise, $\cos A = \sin B = \frac{c}{b}$.

Geometry Activity

Trigonometric Ratios



- Fold a rectangular piece of paper along a diagonal from *A* to *C*. Then cut along the fold to form right triangle *ABC*. Write the name of each angle on the inside of the triangle.
- Fold the triangle so that there are two segments perpendicular to $\overline{BA}$. Label points *D*, *E*, *F*, and *G* as shown. Use a ruler to measure $\overline{AC}$, $\overline{AB}$, $\overline{BC}$, $\overline{AF}$, $\overline{AG}$, $\overline{FG}$, $\overline{AD}$, $\overline{AE}$, and $\overline{DE}$ to the nearest millimeter.

Analyze

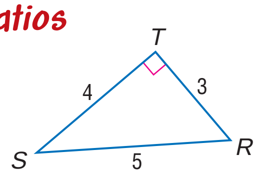
1. What is true of $\triangle AED$, $\triangle AGF$, and $\triangle ABC$?
2. Copy the table. Write the ratio of the side lengths for each trigonometric ratio. Then calculate a value for each ratio to the nearest ten-thousandth.

	In $\triangle AED$	in $\triangle AGF$	In $\triangle ABC$
sin *A*			
cos *A*			
tan *A*			

3. Study the table. Write a sentence about the patterns you observe with the trigonometric ratios.
4. What is true about $m\angle A$ in each triangle?

As the Geometry Activity suggests, the value of a trigonometric ratio depends *only* on the measure of the angle. It does not depend on the size of the triangle.

Example 1 Find Sine, Cosine, and Tangent Ratios



Find sin *R*, cos *R*, tan *R*, sin *S*, cos *S*, and tan *S*. Express each ratio as a fraction and as a decimal.

$\sin R = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5} \text{ or } 0.8$

$\cos R = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5} \text{ or } 0.6$

$\tan R = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{ST}{RT} = \frac{4}{3} \text{ or } 1.\overline{3}$

$\sin S = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{RT}{RS} = \frac{3}{5} \text{ or } 0.6$

$\cos S = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{ST}{RS} = \frac{4}{5} \text{ or } 0.8$

$\tan S = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{RT}{ST} = \frac{3}{4} \text{ or } 0.75$

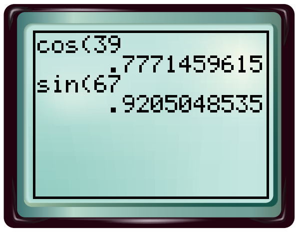
You can use a calculator to evaluate expressions involving trigonometric ratios.

Example 2 Evaluate Expressions

Use a calculator to find each value to the nearest ten thousandth.

a. cos 39°

KEYSTROKES: [COS] 39 [ENTER]



$\cos 39° \approx 0.7771$

b. sin 67°

KEYSTROKES: [SIN] 67 [ENTER]

$\sin 67° \approx 0.9205$

Study Tip

Graphing calculator

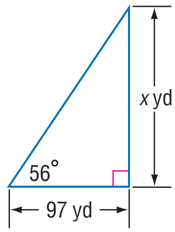
Be sure your calculator is in degree mode rather than radian mode. The [SIN], [COS], and [TAN] keys will calculate the value of the sine, cosine, and tangent of the angle measure. Your calculator may require you to input the angle *before* using the trigonometric key.

USE TRIGONOMETRIC RATIOS You can use trigonometric ratios to find the missing measures of a right triangle if you know the measures of two sides of a triangle or the measure of one side and one acute angle.

Example 3 Use Trigonometric Ratios to Find a Length

SURVEYING **Dakota is standing on the ground 97 yards from the base of a cliff. Using a theodolite, he noted that the angle formed by the ground and the line of sight to the top of the cliff was 56°. Find the height of the cliff to the nearest yard.**

Let x be the height of the cliff in yards.



$\tan 56° = \frac{x}{97}$ $\quad \tan = \frac{\text{leg opposite}}{\text{leg adjacent}}$

$97 \tan 56° = x$ $\quad$ Multiply each side by 97.

Use a calculator to find x.

KEYSTROKES: 97 [TAN] 56 [ENTER] 143.8084139

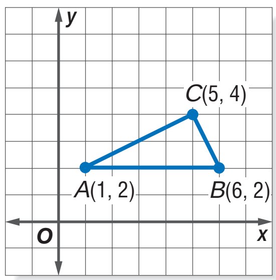
The cliff is about 144 yards high.

When solving equations like $3x = -27$, you use the inverse of multiplication to find x. In trigonometry, you can find the measure of the angle by using the inverse of sine, cosine, or tangent.

Given equation	To find the angle	Read as
$\sin A = x$	$A = \sin^{-1}(x)$	*A* equals *the inverse sine of x*.
$\cos A = y$	$A = \cos^{-1}(y)$	*A* equals *the inverse cosine of y*.
$\tan A = z$	$A = \tan^{-1}(z)$	*A* equals *the inverse tangent of z*.

Example 4 Use Trigonometric Ratios to Find an Angle Measure

COORDINATE GEOMETRY **Find $m\angle A$ in right triangle ABC for $A(1, 2)$, $B(6, 2)$, and $C(5, 4)$.**



Explore You know the coordinates of the vertices of a right triangle and that $\angle C$ is the right angle. You need to find the measure of one of the angles.

Plan Use the Distance Formula to find the measure of each side. Then use one of the trigonometric ratios to write an equation. Use the inverse to find $m\angle A$.

Study Tip

Calculators
The second functions of the SIN, COS, and TAN keys are usually the inverses.

Solve $AB = \sqrt{(6-1)^2 + (2-2)^2}$ $\quad BC = \sqrt{(5-6)^2 + (4-2)^2}$

$= \sqrt{25+0}$ or 5 $\quad = \sqrt{1+4}$ or $\sqrt{5}$

$AC = \sqrt{(5-1)^2 + (4-2)^2}$

$= \sqrt{16+4}$

$= \sqrt{20}$ or $2\sqrt{5}$

Use the cosine ratio.

$\cos A = \frac{AC}{AB}$ $\quad \cos = \frac{\text{leg adjacent}}{\text{hypotenuse}}$

$\cos A = \frac{2\sqrt{5}}{5}$ $\quad AC = 2\sqrt{5}$ and $AB = 5$

$A = \cos^{-1}\left(\frac{2\sqrt{5}}{5}\right)$ $\quad$ Solve for A.

Use a calculator to find $m\angle A$.

KEYSTROKES: [2nd] [COS⁻¹] 2 [2nd] [√] 5 [)] [÷] 5 [ENTER]

$m\angle A \approx 26.56505118$

The measure of $\angle A$ is about 26.6.

Examine Use the sine ratio to check the answer.

$\sin A = \frac{BC}{AB}$ $\quad \sin = \frac{\text{leg opposite}}{\text{hypotenuse}}$

$\sin A = \frac{\sqrt{5}}{5}$ $\quad BC = \sqrt{5}$ and $AB = 5$

KEYSTROKES: [2nd] [SIN⁻¹] [2nd] [√] 5 [)] [÷] 5 [ENTER]

$m\angle A \approx 26.56505118$

The answer is correct.

Check for Understanding

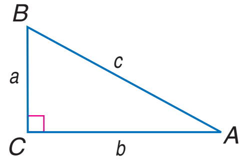
Concept Check

1. **Explain** why trigonometric ratios do not depend on the size of the right triangle.
1. The triangles are similar, so the ratios remain the same.
2. **OPEN ENDED** Draw a right triangle and label the measures of one acute angle and the measure of the side opposite that angle. Then solve for the remaining measures. **2–4. See margin.**
3. **Compare and contrast** the sine, cosine, and tangent ratios.
4. **Explain** the difference between $\tan A = \frac{x}{y}$ and $\tan^{-1}\left(\frac{x}{y}\right) = A$.

Guided Practice

GUIDED PRACTICE KEY	
Exercises	Examples
5, 6	1
7–12	2
17	3
13–16	4

Use $\triangle ABC$ to find $\sin A$, $\cos A$, $\tan A$, $\sin B$, $\cos B$, and $\tan B$. Express each ratio as a fraction and as a decimal to the nearest hundredth. **5–6. See margin.**



5. $a = 14$, $b = 48$, and $c = 50$
6. $a = 8$, $b = 15$, and $c = 17$

Use a calculator to find each value. Round to the nearest ten-thousandth.

7. $\sin 57°$ **0.8387**
8. $\cos 60°$ **0.5000**
9. $\cos 33°$ **0.8387**
10. $\tan 30°$ **0.5774**
11. $\tan 45°$ **1.0000**
12. $\sin 85°$ **0.9962**

Find the measure of each angle to the nearest tenth of a degree.

13. $\tan A = 1.4176$

14. $\sin B = 0.6307$

COORDINATE GEOMETRY **Find the measure of the angle to the nearest tenth in each right triangle *ABC*.**

15. $\angle A$ in $\triangle ABC$, for $A(6, 0)$, $B(-4, 2)$, and $C(0, 6)$

16. $\angle B$ in $\triangle ABC$, for $A(3, -3)$, $B(7, 5)$, and $C(7, -3)$

Application

17. **SURVEYING** Maureen is standing on horizontal ground level with the base of the CN Tower in Toronto, Ontario. The angle formed by the ground and the line segment from her position to the top of the tower is 31.2°. She knows that the height of the tower to the top of the antennae is about 1815 feet. Find her distance from the CN Tower to the nearest foot.

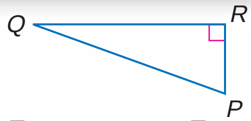
Practice and Apply

Homework Help

For Exercises	See Examples
18–21, 28–36	1
22–27	2
43–48	3
37–42, 52–54	4

Extra Practice
See page 767.

Use $\triangle PQR$ with right angle *R* to find sin *P*, cos *P*, tan *P*, sin *Q*, cos *Q*, and tan *Q*. Express each ratio as a fraction, and as a decimal to the nearest hundredth.



18. $p = 12$, $q = 35$, and $r = 37$

19. $p = \sqrt{6}$, $q = 2\sqrt{3}$, and $r = 3\sqrt{2}$

20. $p = \frac{3}{2}$, $q = \frac{3\sqrt{3}}{2}$, and $r = 3$

21. $p = 2\sqrt{3}$, $q = \sqrt{15}$, and $r = 3\sqrt{3}$

Use a calculator to find each value. Round to the nearest ten-thousandth.

22. sin 6°

23. tan 42.8°

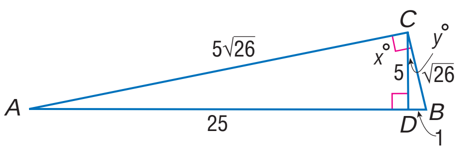
24. cos 77°

25. sin 85.9°

26. tan 12.7°

27. cos 22.5°

Use the figure to find each trigonometric ratio. Express answers as a fraction and as a decimal rounded to the nearest ten-thousandth.



28. sin *A*

29. tan *B*

30. cos *A*

31. sin $x°$

32. cos $x°$

33. tan *A*

34. cos *B*

35. sin $y°$

36. tan $x°$

Find the measure of each angle to the nearest tenth of a degree.

37. $\sin B = 0.7245$

38. $\cos C = 0.2493$

39. $\tan E = 9.4618$

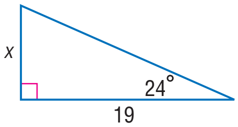
40. $\sin A = 0.4567$

41. $\cos D = 0.1212$

42. $\tan F = 0.4279$

Find *x*. Round to the nearest tenth.

43.



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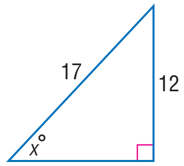

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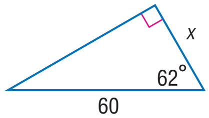

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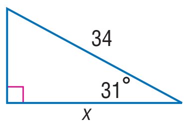

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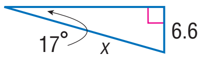

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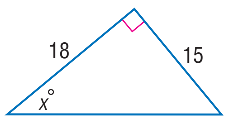
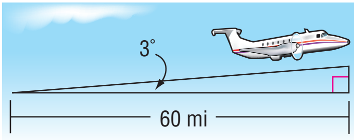
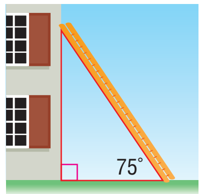


David R. Frazier/Photo Researchers

49. **AVIATION** A plane is one mile above sea level when it begins to climb at a constant angle of 3° for the next 60 ground miles. About how far above sea level is the plane after its climb?



SAFETY **For Exercises 50 and 51, use the following information.**
To guard against a fall, a ladder should make an angle of 75° or less with the ground.



50. What is the maximum height that a 20-foot ladder can reach safely?

51. How far from the building is the base of the ladder at the maximum height?

COORDINATE GEOMETRY **Find the measure of each angle to the nearest tenth in each right triangle.**

52. $\angle J$ in $\triangle JCL$ for $J(2, 2)$, $C(2, -2)$, and $L(7, -2)$

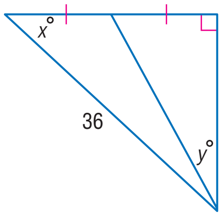
53. $\angle C$ in $\triangle BCD$ for $B(-1, -5)$, $C(-6, -5)$, and $D(-1, 2)$

54. $\angle X$ in $\triangle XYZ$ for $X(-5, 0)$, $Y(7, 0)$, and $Z(0, \sqrt{35})$

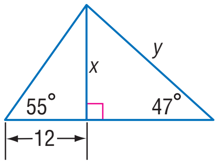
55. Find the perimeter of $\triangle ABC$ if $m\angle A = 35$, $m\angle C = 90$, and $AB = 20$ inches.

Find x and y. Round to the nearest tenth.

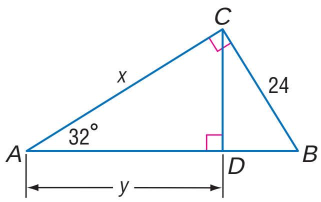
56.



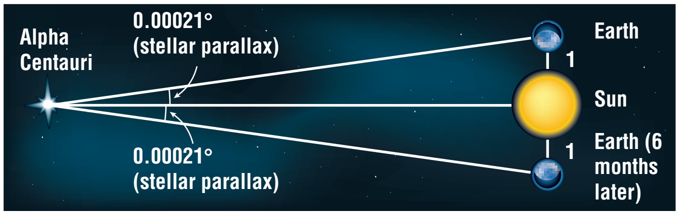
57.



58.



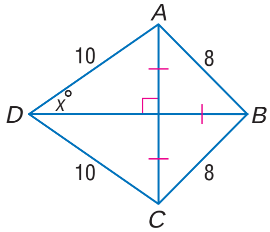
ASTRONOMY **For Exercises 59 and 60, use the following information.**
One way to find the distance between the sun and a relatively close star is to determine the angles of sight for the star exactly six months apart. Half the measure formed by these two angles of sight is called the *stellar parallax*. Distances in space are sometimes measured in *astronomical units*. An astronomical unit is equal to the average distance between Earth and the sun.



59. Find the distance between Alpha Centauri and the sun.

60. Make a conjecture as to why this method is used only for close stars.

61. **CRITICAL THINKING** Use the figure at the right to find sin x°.



Exercise 61

62. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson.

How do surveyors determine angle measures?

Include the following in your answer:
- where theodolites are used, and
- the kind of information one obtains from a theodolite.

More About. . .

Astronomy

The stellar parallax is one of several methods of triangulation used to determine the distance of stars from the sun. Another method is trigonometric parallax, which measures the displacement of a nearby star relative to a more distant star.

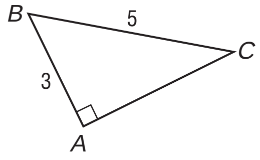
Source: www.infoplease.com

Standardized Test Practice

63. Find cos *C*.

Ⓐ $\frac{3}{5}$ Ⓑ $\frac{3}{4}$

Ⓒ $\frac{4}{5}$ Ⓓ $\frac{5}{4}$

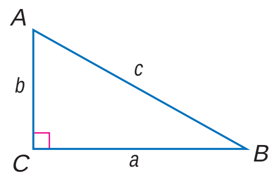


64. ALGEBRA If $x^2 = 15^2 + 24^2 - 15(24)$, find x.

Ⓐ 20.8 Ⓑ 21 Ⓒ 12 Ⓓ 9

Extending the Lesson

Each of the basic trigonometric ratios has a reciprocal ratio. The reciprocals of the sine, cosine, and tangent are called the *cosecant*, *secant*, and the *cotangent*, respectively.



Reciprocal	Trigonometric Ratio	Abbreviation	Definition
$\frac{1}{\sin A}$	cosecant of ∠A	csc A	$\frac{\text{measure of the hypotenuse}}{\text{measure of the leg opposite } \angle A} = \frac{c}{a}$
$\frac{1}{\cos A}$	secant of ∠A	sec A	$\frac{\text{measure of the hypotenuse}}{\text{measure of the leg adjacent } \angle A} = \frac{c}{b}$
$\frac{1}{\tan A}$	cotangent of ∠A	cot A	$\frac{\text{measure of the leg adjacent } \angle A}{\text{measure of the leg opposite } \angle A} = \frac{b}{a}$

Use △*ABC* to find csc *A*, sec *A*, cot *A*, csc *B*, sec *B*, and cot *B*. Express each ratio as a fraction or as a radical in simplest form.

65. $a = 3$, $b = 4$, and $c = 5$

66. $a = 12$, $b = 5$, and $c = 13$

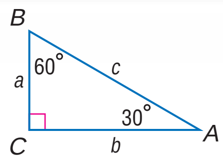
67. $a = 4$, $b = 4\sqrt{3}$, and $c = 8$

68. $a = 2\sqrt{2}$, $b = 2\sqrt{2}$, and $c = 4$

Maintain Your Skills

Mixed Review

Find each measure. *(Lesson 7-3)*



69. If $a = 4$, find b and c.

70. If $b = 3$, find a and c.

71. If $c = 5$, find a and b.

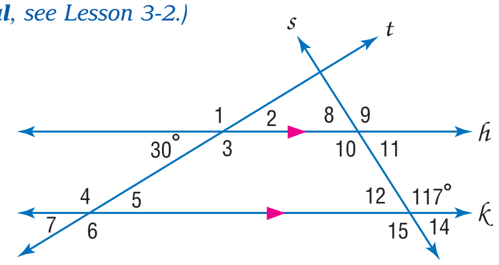
Determine whether each set of measures can be the sides of a right triangle. Then state whether they form a Pythagorean triple. *(Lesson 7-2)*

72. 4, 5, 6 **73.** 5, 12, 13 **74.** 9, 12, 15 **75.** 8, 12, 16

76. TELEVISION During a 30-minute television program, the ratio of minutes of commercials to minutes of the actual show is 4 : 11. How many minutes are spent on commercials? *(Lesson 6-1)*

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Find each angle measure if *h* ∥ *k*.** *(To review* ***angles formed by parallel lines and a transversal****, see Lesson 3-2.)*



77. $m\angle 15$ **78.** $m\angle 7$

79. $m\angle 3$ **80.** $m\angle 12$

81. $m\angle 11$ **82.** $m\angle 4$

7-5 Angles of Elevation and Depression

What You'll Learn

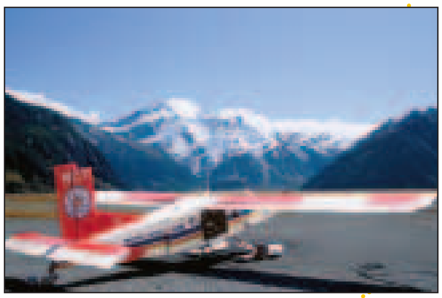
- Solve problems involving angles of elevation.
- Solve problems involving angles of depression.

Vocabulary

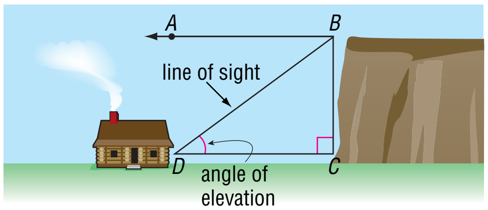
- angle of elevation
- angle of depression

How do airline pilots use angles of elevation and depression?

A pilot is getting ready to take off from Mountain Valley airport. She looks up at the peak of a mountain immediately in front of her. The pilot must estimate the speed needed and the angle formed by a line along the runway and a line from the plane to the peak of the mountain to clear the mountain.



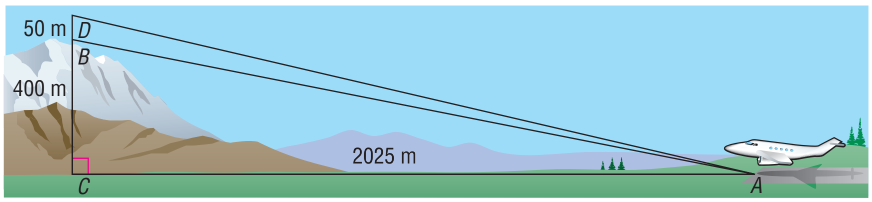
ANGLES OF ELEVATION An **angle of elevation** is the angle between the line of sight and the horizontal when an observer looks upward.



Example 1 Angle of Elevation

AVIATION **The peak of Goose Bay Mountain is 400 meters higher than the end of a local airstrip. The peak rises above a point 2025 meters from the end of the airstrip. A plane takes off from the end of the runway in the direction of the mountain at an angle that is kept constant until the peak has been cleared. If the pilot wants to clear the mountain by 50 meters, what should the angle of elevation be for the takeoff to the nearest tenth of a degree?**

Make a drawing.



Since *CB* is 400 meters and *BD* is 50 meters, *CD* is 450 meters. Let x represent $m\angle DAC$.

$$\tan x^\circ = \frac{CD}{AC} \qquad \tan = \frac{\text{opposite}}{\text{adjacent}}$$

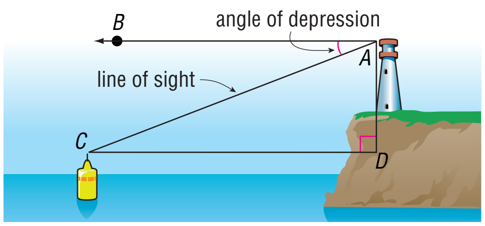
$$\tan x^\circ = \frac{450}{2025} \qquad CD = 450,\ AC = 2025$$

$$x = \tan^{-1}\left(\frac{450}{2025}\right) \qquad \text{Solve for } x.$$

$$x \approx 12.5 \qquad \text{Use a calculator.}$$

The angle of elevation for the takeoff should be more than 12.5°.

ANGLES OF DEPRESSION An **angle of depression** is the angle between the line of sight when an observer looks downward, and the horizontal.

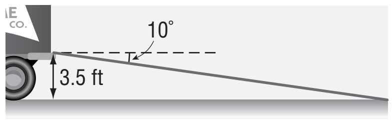


Standardized Test Practice

Example 2 Angle of Depression

Short-Response Test Item

The tailgate of a moving van is 3.5 feet above the ground. A loading ramp is attached to the rear of the van at an incline of 10°. Find the length of the ramp to the nearest tenth foot.

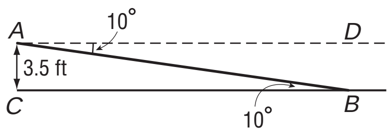


Read the Test Item

The angle of depression between the ramp and the horizontal is 10°. Use trigonometry to find the length of the ramp.

Solve the Test Item

Method 1



The ground and the horizontal level with the back of the van are parallel. Therefore, $m\angle DAB = m\angle ABC$ since they are alternate interior angles.

$$\sin 10° = \frac{3.5}{AB}$$

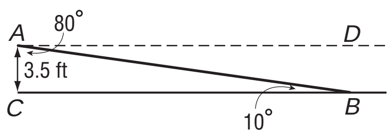
$$AB \sin 10° = 3.5$$

$$AB = \frac{3.5}{\sin 10°}$$

$$AB \approx 20.2$$

The ramp is about 20.2 feet long.

Method 2



The horizontal line from the back of the van and the segment from the ground to the back of the van are perpendicular. So, $\angle DAB$ and $\angle BAC$ are complementary angles. Therefore, $m\angle BAC = 90 - 10$ or 80.

$$\cos 80° = \frac{3.5}{AB}$$

$$AB \cos 80° = 3.5$$

$$AB = \frac{3.5}{\cos 80°}$$

$$AB \approx 20.2$$

The Princeton Review

Test-Taking Tip

Draw a Figure It may be helpful to draw a figure using only information necessary to solve a problem.

Study Tip

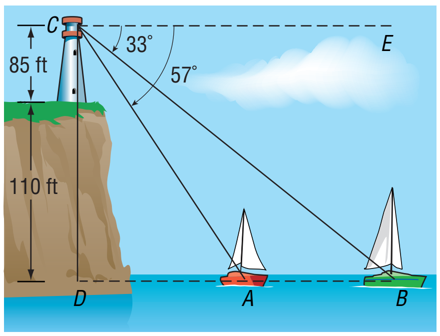
Common Misconception

The angle of depression is often not an angle of the triangle, but the complement to an angle of the triangle. In $\triangle DBC$, the angle of depression is $\angle BCE$, not $\angle DCB$.

Angles of elevation or depression to two different objects can be used to find the distance between those objects.

Example 3 Indirect Measurement

Olivia is in a lighthouse on a cliff. She observes two sailboats due east of the lighthouse. The angles of depression to the two boats are 33° and 57°. Find the distance between the two sailboats to the nearest foot.



$\triangle CDA$ and $\triangle CDB$ are right triangles, and and $CD = 110 + 85$ or 195. The distance between the boats is AB or $BD - AD$. Use the right triangles to find these two lengths.

Study Tip

Look Back

To review **alternate interior angles**, see Lesson 3-2.

Because $\overline{CE}$ and $\overline{DB}$ are horizontal lines, they are parallel. Thus, $\angle ECB \cong \angle CBD$ and $\angle ECA \cong \angle CAD$ because they are alternate interior angles. This means that $m\angle CBD = 33$ and $m\angle CAD = 57$.

$\tan 33° = \frac{195}{DB}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle CBD = 33$
$DB \tan 33° = 195$	Multiply each side by *DB*.
$DB = \frac{195}{\tan 33°}$	Divide each side by tan 33°.
$DB \approx 300.27$	Use a calculator.
$\tan 57° = \frac{195}{DA}$	$\tan = \frac{\text{opposite}}{\text{adjacent}}$; $m\angle CAD = 57$
$DA \tan 57° = 195$	Multiply each side by *DA*.
$DA = \frac{195}{\tan 57°}$	Divide each side by tan 57°.
$DA \approx 126.63$	Use a calculator.

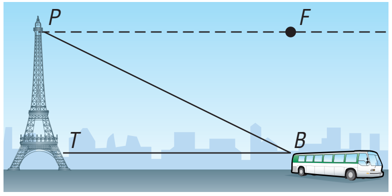
The distance between the boats is $DB - DA$.

$DB - DA \approx 300.27 - 126.63$ or about 174 feet.

Check for Understanding

Concept Check

1. **OPEN ENDED** Find a real-life example of an angle of depression. Draw a diagram and identify the angle of depression.
2. **Explain** why an angle of elevation is given that name.
3. **Name** the angles of depression and elevation in the figure.

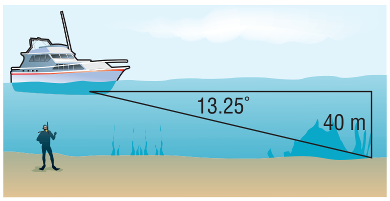


Guided Practice

4. **AVIATION** A pilot is flying at 10,000 feet and wants to take the plane up to 20,000 feet over the next 50 miles. What should be his angle of elevation to the nearest tenth? (*Hint:* There are 5280 feet in a mile.)

5. **SHADOWS** Find the angle of elevation of the sun when a 7.6-meter flagpole casts a 18.2-meter shadow. Round to the nearest tenth of a degree.

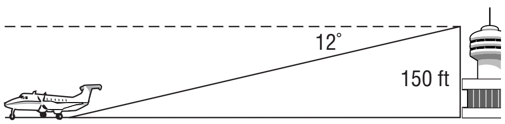
6. **SALVAGE** A salvage ship uses sonar to determine that the angle of depression to a wreck on the ocean floor is 13.25°. The depth chart shows that the ocean floor is 40 meters below the surface. How far must a diver lowered from the salvage ship walk along the ocean floor to reach the wreck?



Standardized Test Practice

A B C D

7. **SHORT RESPONSE** From the top of a 150-foot high tower, an air traffic controller observes an airplane on the runway. To the nearest foot, how far from the base of the tower is the airplane?



Practice and Apply

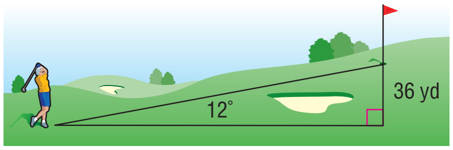
Homework Help

For Exercises	See Examples
12, 14–18	1
9–11	2
8, 13, 19, 20	3

Extra Practice
See page 768.

8. **BOATING** Two boats are observed by a parasailer 75 meters above a lake. The angles of depression are 12.5° and 7°. How far apart are the boats?

9. **GOLF** A golfer is standing at the tee, looking up to the green on a hill. If the tee is 36 yards lower than the green and the angle of elevation from the tee to the hole is 12°, find the distance from the tee to the hole.

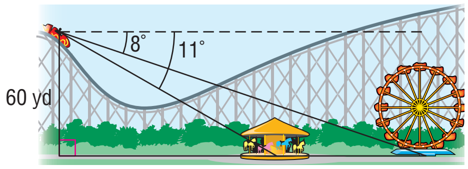


10. **AVIATION** After flying at an altitude of 500 meters, a helicopter starts to descend when its ground distance from the landing pad is 11 kilometers. What is the angle of depression for this part of the flight?

11. **SLEDDING** A sledding run is 300 yards long with a vertical drop of 27.6 yards. Find the angle of depression of the run.

12. **RAILROADS** The Monongahela Incline overlooks the city of Pittsburgh, Pennsylvania. Refer to the information at the left to determine the incline of the railway.

13. **AMUSEMENT PARKS** From the top of a roller coaster, 60 yards above the ground, a rider looks down and sees the merry-go-round and the Ferris wheel. If the angles of depression are 11° and 8° respectively, how far apart are the merry-go-round and the Ferris wheel?



CIVIL ENGINEERING **For Exercises 14 and 15, use the following information.**
The percent grade of a highway is the ratio of the vertical rise or fall over a given horizontal distance. The ratio is expressed as a percent to the nearest whole number. Suppose a highway has a vertical rise of 140 feet for every 2000 feet of horizontal distance.

14. Calculate the percent grade of the highway.

15. Find the angle of elevation that the highway makes with the horizontal.

16. **SKIING** A ski run has an angle of elevation of 24.4° and a vertical drop of 1100 feet. To the nearest foot, how long is the ski run?

GEYSERS **For Exercises 17 and 18, use the following information.**
Kirk visits Yellowstone Park and Old Faithful on a perfect day. His eyes are 6 feet from the ground, and the geyser can reach heights ranging from 90 feet to 184 feet.

17. If Kirk stands 200 feet from the geyser and the eruption rises 175 feet in the air, what is the angle of elevation to the top of the spray to the nearest tenth?

18. In the afternoon, Kirk returns and observes the geyser's spray reach a height of 123 feet when the angle of elevation is 37°. How far from the geyser is Kirk standing to the nearest tenth of a foot?

19. **BIRDWATCHING** Two observers are 200 feet apart, in line with a tree containing a bird's nest. The angles of elevation to the bird's nest are 30° and 60°. How far is each observer from the base of the tree?

More About. . .

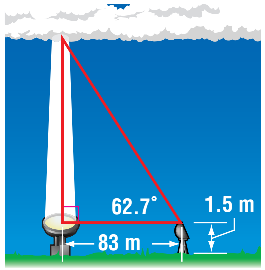
Railroads

The Monongahela Incline is 635 feet long with a vertical rise of 369.39 feet. It was built at a cost of $50,000 and opened on May 28, 1870. It is still used by commuters to and from Pittsburgh.

Source: www.portauthority.org

R. Krubner/H. Armstrong Roberts

20. **METEOROLOGY** The altitude of the base of a cloud formation is called the *ceiling*. To find the ceiling one night, a meteorologist directed a spotlight vertically at the clouds. Using a theodolite placed 83 meters from the spotlight and 1.5 meters above the ground, he found the angle of elevation to be 62.7°. How high was the ceiling?



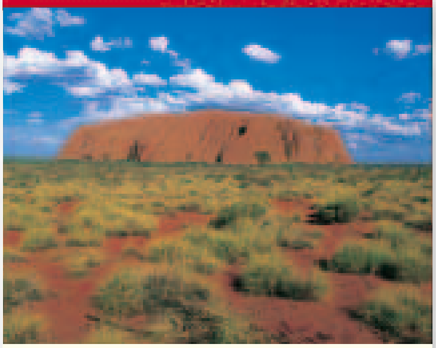
MEDICINE **For Exercises 21–23, use the following information.**
A doctor is using a treadmill to assess the strength of a patient's heart. At the beginning of the exam, the 48-inch long treadmill is set at an incline of 10°.

21. How far off the horizontal is the raised end of the treadmill at the beginning of the exam?

22. During one stage of the exam, the end of the treadmill is 10 inches above the horizontal. What is the incline of the treadmill to the nearest degree?

23. Suppose the exam is divided into five stages and the incline of the treadmill is increased 2° for each stage. Does the end of the treadmill rise the same distance between each stage?

More About. . .



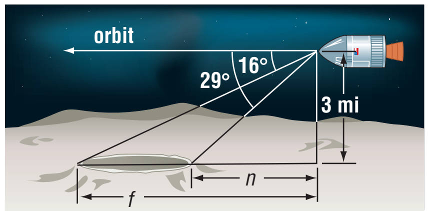
Travel

Ayers Rock is the largest monolith, a type of rock formation, in the world. It is approximately 3.6 kilometers long and 2 kilometers wide. The rock is believed to be the tip of a mountain, two thirds of which is underground.

Source: www.atn.com.au

24. **TRAVEL** Ulura or Ayers Rock is a sacred place for Aborigines of the western desert of Australia. Kwan-Yong uses a theodolite to measure the angle of elevation from the ground to the top of the rock to be 15.85°. He walks half a kilometer closer and measures the angle of elevation to be 25.6°. How high is Ayers Rock to the nearest meter?

25. **AEROSPACE** On July 20, 1969, Neil Armstrong became the first human to walk on the moon. During this mission, the lunar lander *Eagle* traveled aboard *Apollo 11*. Before sending *Eagle* to the surface of the moon, *Apollo 11* orbited the moon three miles above the surface. At one point in the orbit, the onboard guidance system measured the angles of depression to the far and near edges of a large crater. The angles measured 16° and 29°, respectively. Find the distance across the crater.



Online Research **Data Update** Use the Internet to determine the angle of depression formed by someone aboard the international space station looking down to your community. Visit www.geometryonline.com/data_update to learn more.

26. **CRITICAL THINKING** Two weather observation stations are 7 miles apart. A weather balloon is located between the stations. From Station 1, the angle of elevation to the weather balloon is 33°. From Station 2, the angle of elevation to the balloon is 52°. Find the altitude of the balloon to the nearest tenth of a mile.

27. ***WRITING IN MATH*** Answer the question that was posed at the beginning of the lesson.

How do airline pilots use angles of elevation and depression?

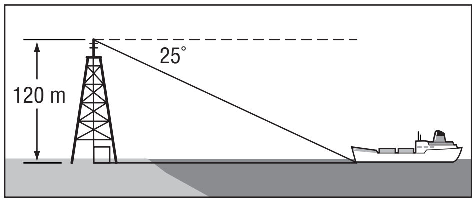
Include the following in your answer:

- when pilots use angles of elevation or depression, and
- the difference between angles of elevation and depression.

John Mead/Science Photo Library/Photo Researchers

Standardized Test Practice

28. The top of a signal tower is 120 meters above sea level. The angle of depression from the top of tower to a passing ship is 25°. How many meters from the foot of the tower is the ship?



(A) 283.9 m (B) 257.3 m

(C) 132.4 m (D) 56 m

29. **ALGEBRA** If $\frac{y}{28} = \frac{x}{16}$, then find x when $y = \frac{1}{2}$.

(A) $\frac{2}{7}$ (B) $\frac{4}{7}$ (C) $\frac{7}{4}$ (D) $3\frac{1}{2}$

Maintain Your Skills

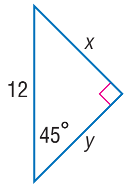
Mixed Review

Find the measure of each angle to the nearest tenth of a degree. *(Lesson 7-4)*

30. $\cos A = 0.6717$
31. $\sin B = 0.5127$
32. $\tan C = 2.1758$
33. $\cos D = 0.3421$
34. $\sin E = 0.1455$
35. $\tan F = 0.3541$

Find x and y. *(Lesson 7-3)*

36.



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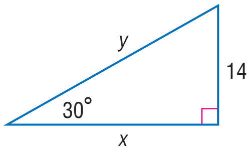
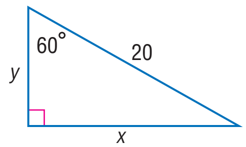


38.

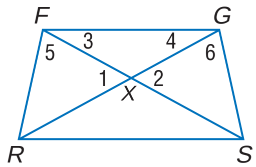


39. **HOBBIES** A twin-engine airplane used for medium-range flights has a length of 78 meters and a wingspan of 90 meters. If a scale model is made with a wingspan of 36 centimeters, find its length. *(Lesson 6-2)*

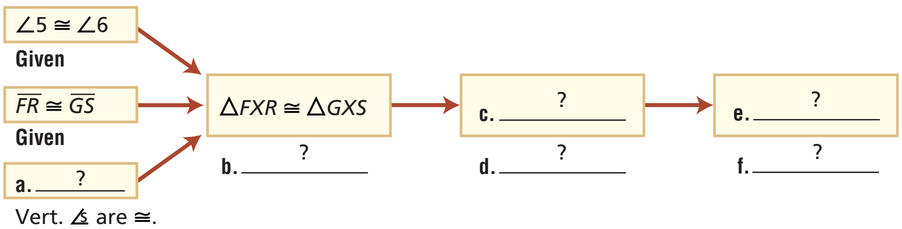
40. Copy and complete the flow proof. *(Lesson 4-6)*

Given: $\angle 5 \cong \angle 6$
$\overline{FR} \cong \overline{GS}$

Prove: $\angle 4 \cong \angle 3$



Proof:



Getting Ready for the Next Lesson

PREREQUISITE SKILL **Solve each proportion.**

*(To review **solving proportions**, see Lesson 6-1.)*

41. $\frac{x}{6} = \frac{35}{42}$
42. $\frac{3}{x} = \frac{5}{45}$
43. $\frac{12}{17} = \frac{24}{x}$
44. $\frac{24}{36} = \frac{x}{15}$
45. $\frac{12}{13} = \frac{48}{x}$
46. $\frac{x}{18} = \frac{5}{8}$
47. $\frac{28}{15} = \frac{7}{x}$
48. $\frac{x}{40} = \frac{3}{26}$

7-6 The Law of Sines

What You'll Learn

- Use the Law of Sines to solve triangles.
- Solve problems by using the Law of Sines.

How are triangles used in radio astronomy?

The Very Large Array (VLA), one of the world's premier astronomical radio observatories, consists of 27 radio antennas in a Y-shaped configuration on the Plains of San Agustin in New Mexico. Astronomers use the VLA to make pictures from the radio waves emitted by astronomical objects. Construction of the antennas is supported by a variety of triangles, many of which are not right triangles.

Vocabulary

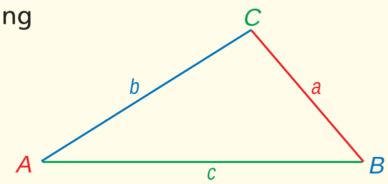
- Law of Sines
- solving a triangle

THE LAW OF SINES In trigonometry, the **Law of Sines** can be used to find missing parts of triangles that are not right triangles.

Key Concept — Law of Sines

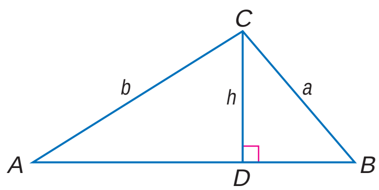
Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of the sides opposite the angles with measures A, B, and C, respectively. Then

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$



Proof — Law of Sines

$\triangle ABC$ is a triangle with an altitude from C that intersects $\overline{AB}$ at D. Let h represent the measure of $\overline{CD}$. Since $\triangle ADC$ and $\triangle BDC$ are right triangles, we can find sin A and sin B.



$\sin A = \frac{h}{b}$ $\quad$ $\sin B = \frac{h}{a}$ $\quad$ Definition of sine

$b \sin A = h$ $\quad$ $a \sin B = h$ $\quad$ Cross products

$b \sin A = a \sin B$ $\quad$ Substitution

$\frac{\sin A}{a} = \frac{\sin B}{b}$ $\quad$ Divide each side by ab.

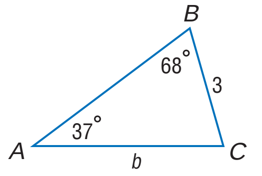
The proof can be completed by using a similar technique with the other altitudes to show that $\frac{\sin A}{a} = \frac{\sin C}{c}$ and $\frac{\sin B}{b} = \frac{\sin C}{c}$.

Roger Ressmeyer/CORBIS

CONTENTS

Example 1 Use the Law of Sines

a. Find *b*. Round to the nearest tenth.



Use the Law of Sines to write a proportion.

$\frac{\sin A}{a} = \frac{\sin B}{b}$ Law of Sines

$\frac{\sin 37°}{3} = \frac{\sin 68°}{b}$ $m\angle A = 37$, $a = 3$, $m\angle B = 68$

$b \sin 37° = 3 \sin 68°$ Cross products

$b = \frac{3 \sin 68°}{\sin 37°}$ Divide each side by sin 37°.

$b \approx 4.6$ Use a calculator.

b. Find $m\angle Z$ to the nearest degree in $\triangle XYZ$ if $y = 17$, $z = 14$, and $m\angle Y = 92$.

Write a proportion relating $\angle Y$, $\angle Z$, y, and z.

$\frac{\sin Y}{y} = \frac{\sin Z}{z}$ Law of Sines

$\frac{\sin 92°}{17} = \frac{\sin Z}{14}$ $m\angle Y = 92$, $y = 17$, $z = 14$

$14 \sin 92° = 17 \sin Z$ Cross products

$\frac{14 \sin 92°}{17} = \sin Z$ Divide each side by 17.

$\sin^{-1}\left(\frac{14 \sin 92°}{17}\right) = Z$ Solve for Z.

$55° \approx Z$ Use a calculator.

So, $m\angle Z \approx 55$.

Study Tip

Rounding
If you round before the final answer, your results may differ from results in which rounding was not done until the final answer.

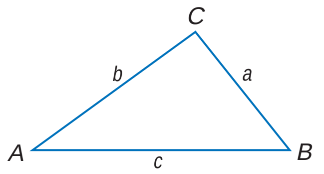
The Law of Sines can be used to solve a triangle. **Solving a triangle** means finding the measures of all of the angles and sides of a triangle.

Example 2 Solve Triangles

Study Tip

Look Back
To review the **Angle Sum Theorem**, see Lesson 4-2.

a. Solve $\triangle ABC$ if $m\angle A = 33$, $m\angle B = 47$, and $b = 14$. Round angle measures to the nearest degree and side measures to the nearest tenth.



We know the measures of two angles of the triangle. Use the Angle Sum Theorem to find $m\angle C$.

$m\angle A + m\angle B + m\angle C = 180$ Angle Sum Theorem

$33 + 47 + m\angle C = 180$ $m\angle A = 33$, $m\angle B = 47$

$80 + m\angle C = 180$ Add.

$m\angle C = 100$ Subtract 80 from each side.

Since we know $m\angle B$ and b, use proportions involving $\frac{\sin B}{b}$.

To find *a*:		**To find *c*:**
$\frac{\sin B}{b} = \frac{\sin A}{a}$	Law of Sines	$\frac{\sin B}{b} = \frac{\sin C}{c}$
$\frac{\sin 47°}{14} = \frac{\sin 33}{a}$	Substitute.	$\frac{\sin 47°}{14} = \frac{\sin 100}{c}$
$a \sin 47° = 14 \sin 33°$	Cross products	$c \sin 47° = 14 \sin 100°$
$a = \frac{14 \sin 33°}{\sin 47°}$	Divide each side by sin 47°.	$c = \frac{14 \sin 100°}{\sin 47°}$
$a \approx 10.4$	Use a calculator.	$c \approx 18.9$

Therefore, $m\angle C = 100$, $a \approx 10.4$, and $c \approx 18.9$.

Study Tip

An Equivalent Proportion
The Law of Sines may also be written as $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$. You may wish to use this form when finding the length of a side.

b. Solve △*ABC* if $m\angle C = 98$, $b = 14$, and $c = 20$. Round angle measures to the nearest degree and side measures to the nearest tenth.

We know the measures of two sides and an angle opposite one of the sides.

$\frac{\sin B}{b} = \frac{\sin C}{c}$ Law of Sines

$\frac{\sin B}{14} = \frac{\sin 98°}{20}$ $m\angle C = 98$, $b = 14$, and $c = 20$

$20 \sin B = 14 \sin 98°$ Cross products

$\sin B = \frac{14 \sin 98°}{20}$ Divide each side by 20.

$B = \sin^{-1}\left(\frac{14 \sin 98°}{20}\right)$ Solve for *B*.

$B \approx 44°$ Use a calculator.

$m\angle A + m\angle B + m\angle C = 180$ Angle Sum Theorem

$m\angle A + 44 + 98 = 180$ $m\angle B = 44$ and $m\angle C = 98$

$m\angle A + 142 = 180$ Add.

$m\angle A = 38$ Subtract 142 from each side.

$\frac{\sin A}{a} = \frac{\sin C}{c}$ Law of Sines

$\frac{\sin 38°}{a} = \frac{\sin 98°}{20}$ $m\angle A = 38$, $m\angle C = 98$, and $c = 20$

$20 \sin 38° = a \sin 98°$ Cross products

$\frac{20 \sin 38°}{\sin 98°} = a$ Divide each side by sin 98°.

$12.4 \approx a$ Use a calculator.

Therefore, $A \approx 38°$, $B \approx 44°$, and $a \approx 12.4$.

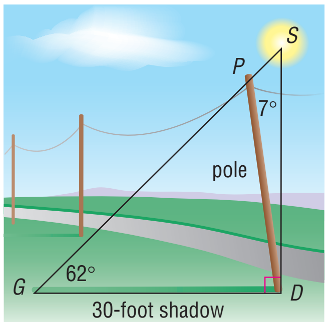
USE THE LAW OF SINES TO SOLVE PROBLEMS

The Law of Sines is very useful in solving direct and indirect measurement applications.

Example 3 Indirect Measurement

When the angle of elevation to the sun is 62°, a telephone pole tilted at an angle of 7° from the vertical casts a shadow of 30 feet long on the ground. Find the length of the telephone pole to the nearest tenth of a foot.

Draw a diagram.



Draw $\overline{SD} \perp \overline{GD}$. Then find the $m\angle GDP$ and $m\angle GPD$.

$m\angle GDP = 90 - 7$ or 83

$m\angle GPD + 62 + 83 = 180$ or $m\angle GPD = 35$

Since you know the measures of two angles of the triangle, $m\angle GDP$ and $m\angle GPD$, and the length of a side opposite one of the angles ($\overline{GD}$ is opposite $\angle GPD$) you can use the Law of Sines to find the length of the pole.

(continued on the next page)

$$\frac{PD}{\sin \angle DGP} = \frac{GD}{\sin \angle GPD}$$ Law of Sines

$$\frac{PD}{\sin 62°} = \frac{30}{\sin 35°}$$ $m\angle DGP = 62$, $m\angle GPD = 35$, and $GD = 30$

$$PD \sin 35° = 30 \sin 62°$$ Cross products

$$PD = \frac{30 \sin 62°}{\sin 35°}$$ Divide each side by sin 35°.

$$PD \approx 46.2$$ Use a calculator.

The telephone pole is about 46.2 feet long.

Concept Summary — *Law of Sines*

The Law of Sines can be used to solve a triangle in the following cases.

Case 1 You know the measures of two angles and any side of a triangle. (AAS or ASA)

Case 2 You know the measures of two sides and an angle opposite one of these sides of the triangle. (SSA)

Check for Understanding

Concept Check

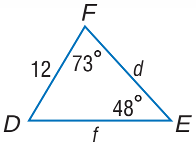
1. **FIND THE ERROR** Makayla and Felipe are trying to find d in $\triangle DEF$.

Makayla

$\sin 59° = \frac{d}{12}$

Felipe

$\frac{\sin 59°}{d} = \frac{\sin 48°}{12}$



Who is correct? Explain your reasoning.

2. **OPEN ENDED** Draw an acute triangle and label the measures of one angle and the lengths of two sides. Explain how to solve the triangle.
3. **Compare** the two cases for the Law of Sines.

Guided Practice

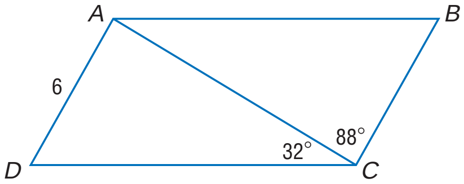
Find each measure using the given measures of $\triangle XYZ$. Round angle measures to the nearest degree and side measures to the nearest tenth.

4. Find y if $x = 3$, $m\angle X = 37$, and $m\angle Y = 68$.
5. Find x if $y = 12.1$, $m\angle X = 57$, and $m\angle Z = 72$.
6. Find $m\angle Y$ if $y = 7$, $z = 11$, and $m\angle Z = 37$.
7. Find $m\angle Z$ if $y = 17$, $z = 14$, and $m\angle Y = 92$.

Solve each $\triangle PQR$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth.

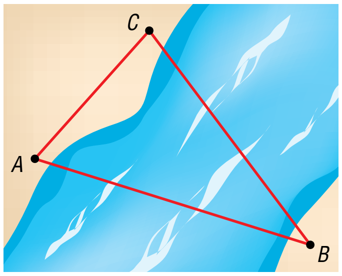
8. $m\angle R = 66$, $m\angle Q = 59$, $p = 72$
9. $p = 32$, $r = 11$, $m\angle P = 105$
10. $m\angle P = 33$, $m\angle R = 58$, $q = 22$
11. $p = 28$, $q = 22$, $m\angle P = 120$
12. $m\angle P = 50$, $m\angle Q = 65$, $p = 12$
13. $q = 17.2$, $r = 9.8$, $m\angle Q = 110.7$

14. Find the perimeter of parallelogram $ABCD$ to the nearest tenth.



Application

15. **SURVEYING** To find the distance between two points A and B that are on opposite sides of a river, a surveyor measures the distance to point C on the same side of the river as point A. The distance from A to C is 240 feet. He then measures the angle from A to B as 62° and measures the angle from C to B as 55°. Find the distance from A to B.



Practice and Apply

Homework Help

For Exercises	See Examples
16–21	1
22–29	2
30–38	3

Extra Practice
See page 768.

Find each measure using the given measures of $\triangle KLM$. Round angle measures to the nearest degree and side measures to the nearest tenth.

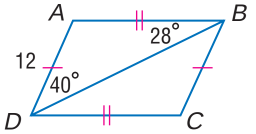
16. If $m\angle L = 45$, $m\angle K = 63$, and $\ell = 22$, find k.
17. If $k = 3.2$, $m\angle L = 52$, and $m\angle K = 70$, find ℓ.
18. If $m = 10.5$, $k = 18.2$, and $m\angle K = 73$, find $m\angle M$.
19. If $k = 10$, $m = 4.8$, and $m\angle K = 96$, find $m\angle M$.
20. If $m\angle L = 88$, $m\angle K = 31$, and $m = 5.4$, find ℓ.
21. If $m\angle M = 59$, $\ell = 8.3$, and $m = 14.8$, find $m\angle L$.

Solve each $\triangle WXY$ described below. Round measures to the nearest tenth.

22. $m\angle Y = 71$, $y = 7.4$, $m\angle X = 41$
23. $x = 10.3$, $y = 23.7$, $m\angle Y = 96$
24. $m\angle X = 25$, $m\angle W = 52$, $y = 15.6$
25. $m\angle Y = 112$, $x = 20$, $y = 56$
26. $m\angle W = 38$, $m\angle Y = 115$, $w = 8.5$
27. $m\angle W = 36$, $m\angle Y = 62$, $w = 3.1$
28. $w = 30$, $y = 9.5$, $m\angle W = 107$
29. $x = 16$, $w = 21$, $m\angle W = 88$

30. An isosceles triangle has a base of 46 centimeters and a vertex angle of 44°. Find the perimeter.

31. Find the perimeter of quadrilateral $ABCD$ to the nearest tenth.

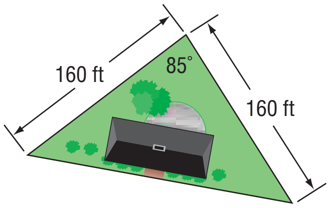


32. **GARDENING** Elena is planning a triangular garden. She wants to build a fence around the garden to keep out the deer. The length of one side of the garden is 26 feet. If the angles at the end of this side are 78° and 44°, find the length of fence needed to enclose the garden.

33. **AVIATION** Two radar stations that are 20 miles apart located a plane at the same time. The first station indicated that the position of the plane made an angle of 43° with the line between the stations. The second station indicated that it made an angle of 48° with the same line. How far is each station from the plane?

34. **SURVEYING** Maria Lopez is a surveyor who must determine the distance across a section of the Rio Grande Gorge in New Mexico. Standing on one side of the ridge, she measures the angle formed by the edge of the ridge and the line of sight to a tree on the other side of the ridge. She then walks along the ridge 315 feet, passing the tree and measures the angle formed by the edge of the ridge and the new line of sight to the same tree. If the first angle is 80° and the second angle is 85°, find the distance across the gorge.

35. **REAL ESTATE** A house is built on a triangular plot of land. Two sides of the plot are 160 feet long, and they meet at an angle of 85°. If a fence is to be placed along the perimeter of the property, how much fencing material is needed?



MIRRORS **For Exercises 36 and 37, use the following information.**
Kayla, Jenna, and Paige live in a town nicknamed "Perpendicular City" because the planners and builders took great care to have all the streets oriented north-south or east-west. The three of them play a game where they signal each other using mirrors. Kayla and Jenna signal each other from a distance of 1433 meters. Jenna turns 27° to signal Paige. Kayla rotates 40° to signal Paige.

36. To the nearest tenth of a meter, how far apart are Kayla and Paige?

37. To the nearest tenth of a meter, how far apart are Jenna and Paige?

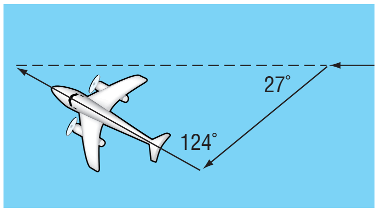
More About...

Aviation

From January 2001 to May 2001, Polly Vacher flew over 29,000 miles to become the first woman to fly the smallest single-engined aircraft around the world via Australia and the Pacific.

Source: www.worldwings.org

AVIATION **For Exercises 38 and 39, use the following information.**
Keisha Jefferson is flying a small plane due west. To avoid the jet stream, she must change her course. She turns the plane 27° to the south and flies 60 miles. Then she makes a turn of 124° heads back to her original course.



38. How far must she fly after the second turn to return to the original course?

39. How many miles did she add to the flight by changing course?

40. **CRITICAL THINKING** Does the Law of Sines apply to the acute angles of a right triangle? Explain your answer.

41. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are triangles used in radio astronomy?

Include the following in your answer:

- a description of what the VLA is and the purpose of the VLA, and
- the purpose of the triangles in the VLA.

Standardized Test Practice

42. **SHORT RESPONSE** In $\triangle XYZ$, if $x = 12$, $m\angle X = 48$, and $m\angle Y = 112$, solve the triangle to the nearest tenth.

43. **ALGEBRA** The table below shows the customer ratings of five restaurants in the *Metro City Guide to Restaurants*. The rating scale is from 1, the worst, to 10, the best. Which of the five restaurants has the best average rating?

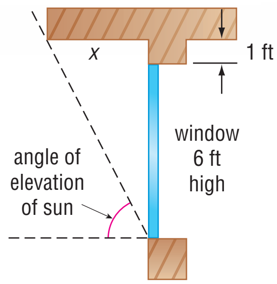
Restaurant	Food	Decor	Service	Value
Del Blanco's	7	9	4	7
Aquavent	8	9	4	6
Le Circus	10	8	3	5
Sushi Mambo	7	7	5	6
Metropolis Grill	9	8	7	7

(A) Metropolis Grill
(B) Le Circus
(C) Aquavent
(D) Del Blanco's

Maintain Your Skills

Mixed Review

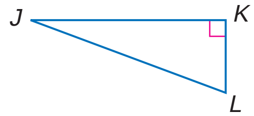
ARCHITECTURE **For Exercises 44 and 45, use the following information.**
Mr. Martinez is an architect who designs houses so that the windows receive minimum sun in the summer and maximum sun in the winter. For Columbus, Ohio, the angle of elevation of the sun at noon on the longest day is 73.5° and on the shortest day is 26.5°. Suppose a house is designed with a south-facing window that is 6 feet tall. The top of the window is to be installed 1 foot below the overhang. *(Lesson 7-5)*



44. How long should the architect make the overhang so that the window gets no direct sunlight at noon on the longest day?

45. Using the overhang from Exercise 44, how much of the window will get direct sunlight at noon on the shortest day?

Use $\triangle JKL$ to find sin J, cos J, tan J, sin L, cos L, and tan L. Express each ratio as a fraction and as a decimal to the nearest hundredth. *(Lesson 7-4)*



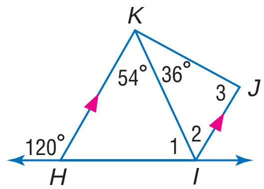
46. $j = 8, k = 17, \ell = 15$

47. $j = 20, k = 29, \ell = 21$

48. $j = 12, k = 24, \ell = 12\sqrt{3}$

49. $j = 7\sqrt{2}, k = 14, \ell = 7\sqrt{2}$

If $\overline{KH}$ is parallel to $\overline{JI}$, find the measure of each angle. *(Lesson 4-2)*



50. $\angle 1$

51. $\angle 2$

52. $\angle 3$

Getting Ready for the Next Lesson

PREREQUISITE SKILL **Evaluate $\frac{c^2 - a^2 - b^2}{-2ab}$ for the given values of a, b, and c.**
*(To review **evaluating expressions**, see page 736.)*

53. $a = 7, b = 8, c = 10$

54. $a = 4, b = 9, c = 6$

55. $a = 5, b = 8, c = 10$

56. $a = 16, b = 4, c = 13$

57. $a = 3, b = 10, c = 9$

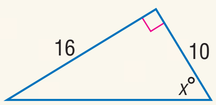
58. $a = 5, b = 7, c = 11$

Practice Quiz 2

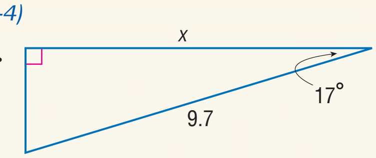
Lessons 7-4 through 7-6

Find x to the nearest tenth. *(Lesson 7-4)*

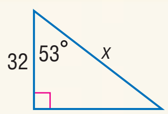
1.



2.

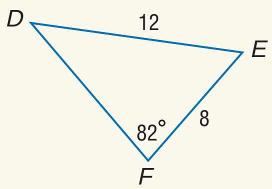


3.



4. COMMUNICATIONS To secure a 500-meter radio tower against high winds, guy wires are attached to the tower 5 meters from the top. The wires form a 15° angle with the tower. Find the distance from the centerline of the tower to the anchor point of the wires. *(Lesson 7-5)*

5. Solve $\triangle DEF$. *(Lesson 7-6)*



Geometry Software Investigation

A Follow-Up of Lesson 7-6

The Ambiguous Case of the Law of Sines

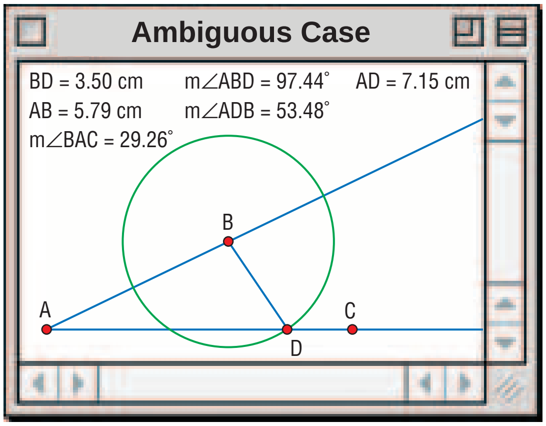
In Lesson 7-6, you learned that you could solve a triangle using the Law of Sines if you know the measures of two angles and any side of the triangle (AAS or ASA). You can also solve a triangle by the Law of Sines if you know the measures of two sides and an angle opposite one of the sides (SSA). When you use SSA to solve a triangle, and the given angle is acute, sometimes it is possible to find two different triangles. You can use The Geometer's Sketchpad to explore this case, called the **ambiguous case**, of the Law of Sines.

Model

Step 1 Construct $\overrightarrow{AB}$ and $\overrightarrow{AC}$. Construct a circle whose center is B so that it intersects $\overrightarrow{AC}$ at two points. Then, construct any radius $\overline{BD}$.

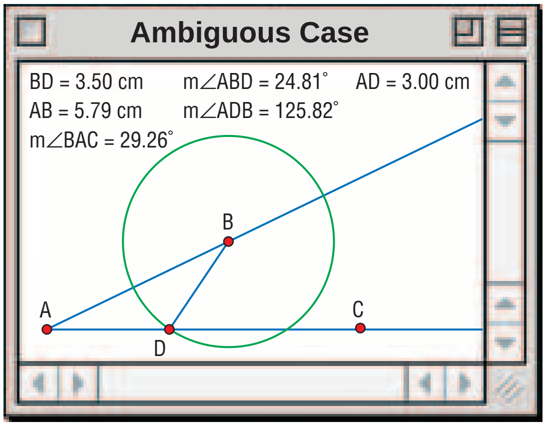
Step 2 Find the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$.

Step 3 Use the rotate tool to move D so that it lies on one of the intersection points of circle B and $\overrightarrow{AC}$. In $\triangle ABD$, find the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$.



Step 4 Using the rotate tool, move D to the other intersection point of circle B and $\overrightarrow{AC}$.

Step 5 Note the measures of $\angle ABD$, $\angle BDA$, and $\overline{AD}$ in $\triangle ABD$.



Analyze

1. Which measures are the same in both triangles?
2. Repeat the activity using different measures for $\angle A$, $\overline{BD}$, and $\overline{AB}$. How do the results compare to the earlier results?

Make a Conjecture

3. Compare your results with those of your classmates. How do the results compare?
4. What would have to be true about circle B in order for there to be one unique solution? Test your conjecture by repeating the activity.
5. Is it possible, given the measures of $\overline{BD}$, $\overline{AB}$, and $\angle A$, to have no solution? Test your conjecture and explain.

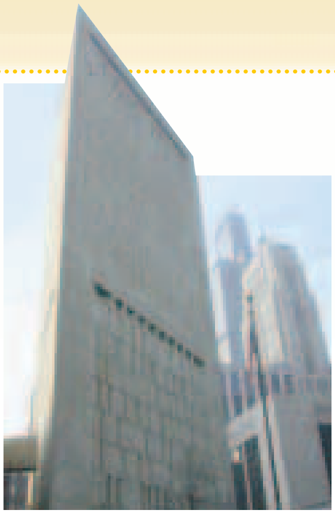
7-7 The Law of Cosines

What You'll Learn

- Use the Law of Cosines to solve triangles.
- Solve problems by using the Law of Cosines.

How are triangles used in building design?

The Chicago Metropolitan Correctional Center is a 27-story triangular federal detention center. The cells are arranged around a lounge-like common area. The architect found that a triangular floor plan allowed for the maximum number of cells to be most efficiently centered around the lounge.



Vocabulary

- Law of Cosines

THE LAW OF COSINES Suppose you know the lengths of the sides of the triangular building and want to solve the triangle. The **Law of Cosines** allows us to solve a triangle when the Law of Sines cannot be used.

Study Tip

Side and Angle
Note that the letter of the side length on the left-hand side of each equation corresponds to the angle measure used with the cosine.

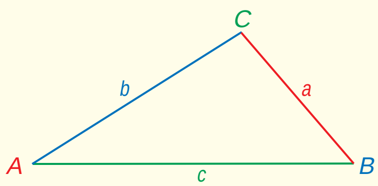
Key Concept — Law of Cosines

Let $\triangle ABC$ be any triangle with a, b, and c representing the measures of sides opposite angles with measures A, B, and C, respectively. Then the following equations are true.

$a^2 = b^2 + c^2 - 2bc \cos A$

$b^2 = a^2 + c^2 - 2ac \cos B$

$c^2 = a^2 + b^2 - 2ab \cos C$

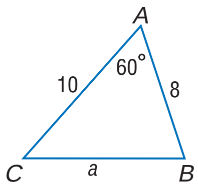


The Law of Cosines can be used to find missing measures in a triangle if you know the measures of two sides and their included angle.

Example 1 Two Sides and the Included Angle

Find a if $c = 8$, $b = 10$, and $m\angle A = 60$.

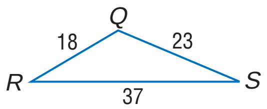
Use the Law of Cosines since the measures of two sides and the included are known.



$a^2 = b^2 + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 10^2 + 8^2 - 2(10)(8) \cos 60°$	$b = 10$, $c = 8$, and $m\angle A = 60$
$a^2 = 164 - 160 \cos 60°$	Simplify.
$a = \sqrt{164 - 160 \cos 60°}$	Take the square root of each side.
$a \approx 9.2$	Use a calculator.

You can also use the Law of Cosines to find the measures of angles of a triangle when you know the measures of the three sides.

Example 2 Three Sides



Find $m\angle R$.

$r^2 = q^2 + s^2 - 2qs \cos R$	Law of Cosines
$23^2 = 37^2 + 18^2 - 2(37)(18) \cos R$	$r = 23$, $q = 37$, $s = 18$
$529 = 1693 - 1332 \cos R$	Simplify.
$-1164 = -1332 \cos R$	Subtract 1693 from each side.
$\frac{-1164}{-1332} = \cos R$	Divide each side by −1332.
$R = \cos^{-1}\left(\frac{1164}{1332}\right)$	Solve for R.
$R \approx 29.1°$	Use a calculator.

USE THE LAW OF COSINES TO SOLVE PROBLEMS Most problems can be solved using more than one method. Choosing the most efficient way to solve a problem is sometimes not obvious.

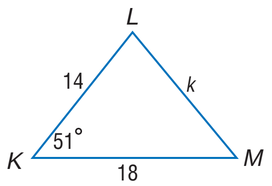
When solving right triangles, you can use sine, cosine, or tangent ratios. When solving other triangles, you can use the Law of Sines or the Law of Cosines. You must decide how to solve each problem depending on the given information.

Example 3 Select a Strategy

Study Tip

Law of Cosines
If you use the Law of Cosines to find another measure, your answer may differ slightly from one found using the Law of Sines. This is due to rounding.

Solve $\triangle KLM$. Round angle measure to the nearest degree and side measure to the nearest tenth.



We do not know whether $\triangle KLM$ is a right triangle, so we must use the Law of Cosines or the Law of Sines. We know the measures of two sides and the included angle. This is SAS, so use the Law of Cosines.

$k^2 = \ell^2 + m^2 - 2\ell m \cos K$	Law of Cosines
$k^2 = 18^2 + 14^2 - 2(18)(14) \cos 51°$	$\ell = 18$, $m = 14$, and $m\angle K = 51$
$k = \sqrt{18^2 + 14^2 - 2(18)(14) \cos 51°}$	Take the square root of each side.
$k \approx 14.2$	Use a calculator.

Next, we can find $m\angle L$ or $m\angle M$. If we decide to find $m\angle L$, we can use either the Law of Sines or the Law of Cosines to find this value. In this case, we will use the Law of Sines.

$\frac{\sin L}{\ell} = \frac{\sin K}{k}$	Law of Sines
$\frac{\sin L}{18} \approx \frac{\sin 51°}{14.2}$	$\ell = 18$, $k \approx 14.2$, and $m\angle K = 51$
$14.2 \sin L \approx 18 \sin 51°$	Cross products
$\sin L \approx \frac{18 \sin 51°}{14.2}$	Divide each side by 14.2.
$L \approx \sin^{-1}\left(\frac{18 \sin 51°}{14.2}\right)$	Take the inverse sine of each side.
$L \approx 80°$	Use a calculator.

Use the Angle Sum Theorem to find $m\angle M$.

$m\angle K + m\angle L + m\angle M = 180$ — Angle Sum Theorem

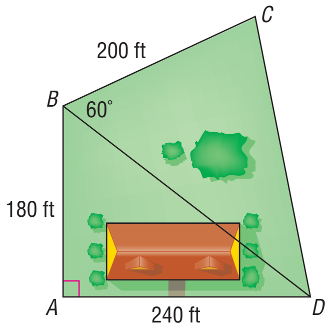
$51 + 80 + m\angle M \approx 180$ — $m\angle K = 51$ and $m\angle L \approx 80$

$m\angle M \approx 49$ — Subtract 131 from each side.

Therefore, $k \approx 14.2$, $m\angle K \approx 80$, and $m\angle M \approx 49$.

Example 4 Use Law of Cosines to Solve Problems

REAL ESTATE **Ms. Jenkins is buying some property that is shaped like quadrilateral *ABCD*. Find the perimeter of the property.**



Use the Pythagorean Theorem to find BD in $\triangle ABD$.

$(AB)^2 + (AD)^2 = (BD)^2$ — Pythagorean Theorem

$180^2 + 240^2 = (BD)^2$ — $AB = 180$, $AD = 240$

$90{,}000 = (BD)^2$ — Simplify.

$300 = BD$ — Take the square root of each side.

Next, use the Law of Cosines to find CD in $\triangle BCD$.

$(CD)^2 = (BC)^2 + (BD)^2 - 2(BC)(BD) \cos \angle CBD$ — Law of Cosines

$(CD)^2 = 200^2 + 300^2 - 2(200)(300) \cos 60°$ — $BC = 200$, $BD = 300$, $m\angle CBD = 60$

$(CD)^2 = 130{,}000 - 120{,}000 \cos 60°$ — Simplify.

$CD = \sqrt{130{,}000 - 120{,}000 \cos 60°}$ — Take the square root of each side.

$CD \approx 264.6$ — Use a calculator.

The perimeter of the property is $180 + 200 + 264.6 + 240$ or about 884.6 feet.

Check for Understanding

Concept Check

1. **OPEN ENDED** Draw and label one acute and one obtuse triangle, illustrating when you can use the Law of Cosines to find the missing measures.
2. **Explain** when you should use the Law of Sines or the Law of Cosines to solve a triangle.
3. **Find a counterexample** for the following statement.
   The Law of Cosines can be used to find the length of a missing side in any triangle.

Guided Practice

In $\triangle BCD$, given the following measures, find the measure of the missing side.

4. $c = \sqrt{2}$, $d = 5$, $m\angle B = 45$
5. $b = 107$, $c = 94$, $m\angle D = 105$

In $\triangle RST$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

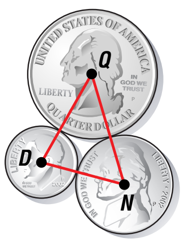
6. $r = 33$, $s = 65$, $t = 56$; $m\angle S$
7. $r = 2.2$, $s = 1.3$, $t = 1.6$; $m\angle R$

Solve each triangle using the given information. Round angle measure to the nearest degree and side measure to the nearest tenth.

8. $\triangle XYZ$: $x = 5$, $y = 10$, $z = 13$
9. $\triangle KLM$: $k = 20$, $m = 24$, $m\angle L = 47$

Application

10. **CRAFTS** Jamie, age 25, is creating a logo for herself and two cousins, ages 10 and 5. She is using a quarter (25 cents), a dime (10 cents), and a nickel (5 cents) to represent their ages. She will hold the coins together by soldering a triangular piece of silver wire so that the three vertices of the triangle lie at the centers of the three circular coins. The diameter of the quarter is 24 millimeters, the diameter of the nickel is 22 millimeters, and the diameter of a dime is 10 millimeters. Find the measures of the three angles in the triangle.



Practice and Apply

Homework Help

For Exercises	See Examples
11–14	1
15–18, 38	2
22–37, 39–41	3

Extra Practice
See page 768.

In $\triangle TUV$, given the following measures, find the measure of the missing side.

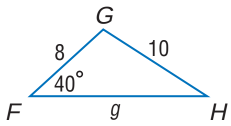
11. $t = 9.1, v = 8.3, m\angle U = 32$
12. $t = 11, u = 17, m\angle V = 78$
13. $u = 11, v = 17, m\angle T = 105$
14. $v = 11, u = 17, m\angle T = 59$

In $\triangle EFG$, given the lengths of the sides, find the measure of the stated angle to the nearest degree.

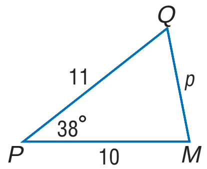
15. $e = 9.1, f = 8.3, g = 16.7; m\angle F$
16. $e = 14, f = 19, g = 32; m\angle E$
17. $e = 325, f = 198, g = 208; m\angle F$
18. $e = 21.9, f = 18.9, g = 10; m\angle G$

Solve each triangle using the given information. Round angle measures to the nearest degree and side measures to the nearest tenth.

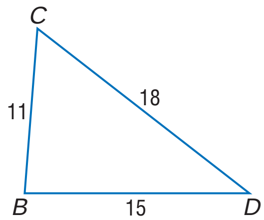
19.



20.

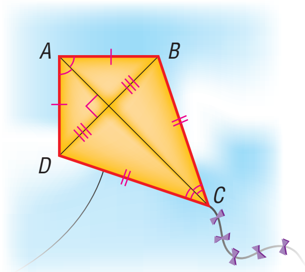


21.



22. $\triangle ABC$: $m\angle A = 42, m\angle C = 77, c = 6$
23. $\triangle ABC$: $a = 10.3, b = 9.5, m\angle C = 37$
24. $\triangle ABC$: $a = 15, b = 19, c = 28$
25. $\triangle ABC$: $m\angle A = 53, m\angle C = 28, c = 14.9$

26. **KITES** Beth is building a kite like the one at the right. If $\overline{AB}$ is 5 feet long, $\overline{BC}$ is 8 feet long, and $\overline{BD}$ is $7\frac{2}{3}$ feet long, find the measure of the angle between the short sides and the angle between the long sides to the nearest degree.

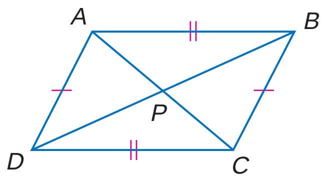


Solve each $\triangle LMN$ described below. Round measures to the nearest tenth.

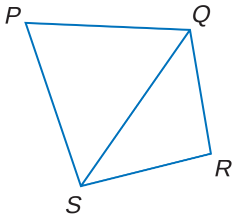
27. $m = 44, \ell = 54, m\angle L = 23$
28. $m = 18, \ell = 24, n = 30$
29. $m = 19, n = 28, m\angle L = 49$
30. $m\angle M = 46, m\angle L = 55, n = 16$
31. $m = 256, \ell = 423, n = 288$
32. $m\angle M = 55, \ell = 6.3, n = 6.7$
33. $m\angle M = 27, \ell = 5, n = 10$
34. $n = 17, m = 20, \ell = 14$
35. $\ell = 14, n = 21, m\angle M = 60$
36. $\ell = 14, m = 15, n = 16$
37. $m\angle L = 51, \ell = 40, n = 35$
38. $\ell = 10, m = 11, n = 12$

CONTENTS

39. In quadrilateral $ABCD$, $AC = 188$, $BD = 214$, $m\angle BPC = 70$, and P is the midpoint of $\overline{AC}$ and $\overline{BD}$. Find the perimeter of quadrilateral $ABCD$.



40. In quadrilateral $PQRS$, $PQ = 721$, $QR = 547$, $RS = 593$, $PS = 756$, and $m\angle P = 58$. Find QS, $m\angle PQS$, and $m\angle R$.



More About. . .

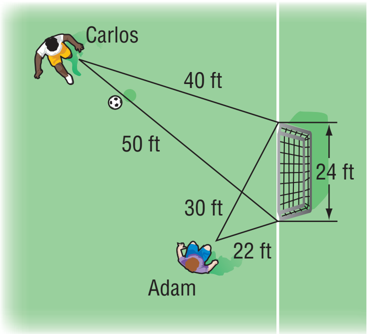
Buildings

The Swissôtel in Chicago, Illinois, is built in the shape of a triangular prism. The lengths of the sides of the triangle are 180 feet, 186 feet, and 174 feet.

Source: Swissôtel

41. **BUILDINGS** Refer to the information at the left. Find the measures of the angles of the triangular building to the nearest tenth.

42. **SOCCER** Carlos and Adam are playing soccer. Carlos is standing 40 feet from one post of the goal and 50 feet from the other post. Adam is standing 30 feet from one post of the goal and 22 feet from the other post. If the goal is 24 feet wide, which player has a greater angle to make a shot on goal?

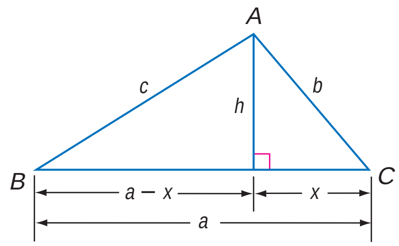


43. **PROOF** Justify each statement for the derivation of the Law of Cosines.

Given: $\overline{AD}$ is an altitude of $\triangle ABC$.

Prove: $c^2 = a^2 + b^2 - 2ab \cos C$

Proof:



Statement	Reasons
a. $c^2 = (a - x)^2 + h^2$	**a.** ?
b. $c^2 = a^2 - 2ax + x^2 + h^2$	**b.** ?
c. $x^2 + h^2 = b^2$	**c.** ?
d. $c^2 = a^2 - 2ax + b^2$	**d.** ?
e. $\cos C = \frac{x}{b}$	**e.** ?
f. $b \cos C = x$	**f.** ?
g. $c^2 = a^2 - 2a(b \cos C) + b^2$	**g.** ?
h. $c^2 = a^2 + b^2 - 2ab \cos C$	**h.** ?

44. **CRITICAL THINKING** Graph $A(-6, -8)$, $B(10, -4)$, $C(6, 8)$, and $D(5, 11)$ on a coordinate plane. Find the measure of interior angle ABC and the measure of exterior angle DCA.

45. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How are triangles used in building construction?

Include the following in your answer:

- why the building was triangular instead of rectangular, and
- why the Law of Sines could not be used to solve the triangle.

Standardized Test Practice

46. For $\triangle DEF$, find d to the nearest tenth if $e = 12$, $f = 15$, and $m\angle D = 75$.

Ⓐ 18.9 Ⓑ 16.6 Ⓒ 15.4 Ⓓ 9.8

47. **ALGEBRA** Ms. LaHue earns a monthly base salary of \$1280 plus a commission of 12.5% of her total monthly sales. At the end of one month, Ms. LaHue earned \$4455. What were her total sales for the month?

Ⓐ \$3175 Ⓑ \$10,240 Ⓒ \$25,400 Ⓓ \$35,640

Maintain Your Skills

Mixed Review

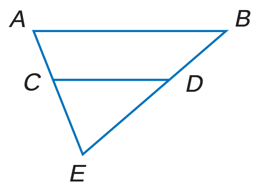
Find each measure using the given measures from $\triangle XYZ$. Round angle measure to the nearest degree and side measure to the nearest tenth. *(Lesson 7-6)*

48. If $y = 4.7$, $m\angle X = 22$, and $m\angle Y = 49$, find x.

49. If $y = 10$, $x = 14$, and $m\angle X = 50$, find $m\angle Y$.

50. **SURVEYING** A surveyor is 100 meters from a building and finds that the angle of elevation to the top of the building is 23°. If the surveyor's eye level is 1.55 meters above the ground, find the height of the building. *(Lesson 7-5)*

For Exercises 51–54, determine whether $\overline{AB} \parallel \overline{CD}$. *(Lesson 6-4)*



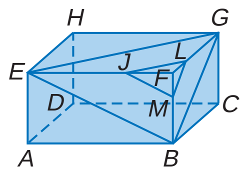
51. $AC = 8.4$, $BD = 6.3$, $DE = 4.5$, and $CE = 6$

52. $AC = 7$, $BD = 10.5$, $BE = 22.5$, and $AE = 15$

53. $AB = 8$, $AE = 9$, $CD = 4$, and $CE = 4$

54. $AB = 5.4$, $BE = 18$, $CD = 3$, and $DE = 10$

Use the figure at the right to write a paragraph proof. *(Lesson 6-3)*



55. **Given:** $\triangle JFM \sim \triangle EFB$
$\triangle LFM \sim \triangle GFB$

Prove: $\triangle JFL \sim \triangle EFG$

56. **Given:** $\overline{JM} \parallel \overline{EB}$
$\overline{LM} \parallel \overline{GB}$

Prove: $\overline{JL} \parallel \overline{EG}$

COORDINATE GEOMETRY **The vertices of $\triangle XYZ$ are $X(8, 0)$, $Y(-4, 8)$, and $Z(0, 12)$. Find the coordinates of the points of concurrency of $\triangle XYZ$ to the nearest tenth.** *(Lesson 5-1)*

57. orthocenter 58. centroid 59. circumcenter

WebQuest Internet Project

Who is Behind This Geometry Idea Anyway?

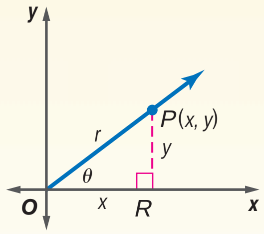
It's time to complete your project. Use the information and data you have gathered about your research topic, two mathematicians, and a geometry problem to prepare a portfolio or Web page. Be sure to include illustrations and/or tables in the presentation.

www.geometryonline.com/webquest

Trigonometric Identities

In algebra, the equation $2(x + 2) = 2x + 4$ is called an *identity* because the equation is true for all values of x. There are equations involving trigonometric ratios that are true for all values of the angle measure. These are called **trigonometric identities**.

In the figure, $P(x, y)$ is in Quadrant I. The Greek letter theta (pronounced THAY tuh) θ, represents the measure of the angle formed by the x-axis and $\overrightarrow{OP}$. Triangle POR is a right triangle. Let r represent the length of the hypotenuse. Then the following are true.



$\sin \theta = \frac{y}{r}$ $\qquad$ $\cos \theta = \frac{x}{r}$ $\qquad$ $\tan \theta = \frac{y}{x}$

$\csc \theta = \frac{r}{y}$ $\qquad$ $\sec \theta = \frac{r}{x}$ $\qquad$ $\cot \theta = \frac{x}{y}$

Notice that $\frac{1}{\sin \theta} = \frac{1}{\frac{y}{r}}$ ⇨ $\frac{1}{\frac{y}{r}} = 1 \div \frac{y}{r}$ ⇨ $1 \div \frac{y}{r} = 1 \cdot \frac{r}{y}$ or $\frac{r}{y}$ ⇨ $\frac{r}{y} = \csc \theta$.

So, $\frac{1}{\sin \theta} = \csc \theta$. This is known as one of the **reciprocal identities**.

Activity

Verify that $\cos^2 \theta + \sin^2 \theta = 1$.

The expression $\cos^2 \theta$ means $(\cos \theta)^2$. To verify an identity, work on only one side of the equation and use what you know to show how that side is equivalent to the other side.

$\cos^2 \theta + \sin^2 \theta \stackrel{?}{=} 1$	Original equation
$\left(\frac{x}{r}\right)^2 + \left(\frac{y}{r}\right)^2 \stackrel{?}{=} 1$	Substitute.
$\frac{x^2}{r^2} + \frac{y^2}{r^2} \stackrel{?}{=} 1$	Simplify.
$\frac{x^2 + y^2}{r^2} \stackrel{?}{=} 1$	Combine fractions with like denominators.
$\frac{r^2}{r^2} \stackrel{?}{=} 1$	Pythagorean Theorem: $x^2 + y^2 = r^2$
$1 = 1$ ✓	Simplify.

Since $1 = 1$, $\cos^2 \theta + \sin^2 \theta = 1$.

Analyze

1. The identity $\cos^2 \theta + \sin^2 \theta = 1$ is known as a **Pythagorean identity**. Why do you think the word *Pythagorean* is used to name this?
2. Find two more reciprocal identities involving $\frac{1}{\cos \theta}$ and $\frac{1}{\tan \theta}$.

Verify each identity.

3. $\frac{\sin \theta}{\cos \theta} = \tan \theta$
4. $\cot \theta = \frac{\cos \theta}{\sin \theta}$
5. $\tan^2 \theta + 1 = \sec^2 \theta$
6. $\cot^2 \theta + 1 = \csc^2 \theta$

Chapter 7 Study Guide and Review

Vocabulary and Concept Check

ambiguous case (p. 384)
angle of depression (p. 372)
angle of elevation (p. 371)
cosine (p. 364)
geometric mean (p. 342)
Law of Cosines (p. 385)
Law of Sines (p. 377)
Pythagorean identity (p. 391)
Pythagorean triple (p. 352)
reciprocal identities (p. 391)
sine (p. 364)
solving a triangle (p. 378)
tangent (p. 364)
trigonometric identity (p. 391)
trigonometric ratio (p. 364)
trigonometry (p. 364)

A complete list of postulates and theorems can be found on pages R1–R8.

Exercises **State whether each statement is *true* or *false*. If false, replace the underlined word or words to make a true sentence.**

1. The Law of Sines can be applied if you know the measures of two sides and an angle <u>opposite</u> one of these sides of the triangle.
2. The tangent of $\angle A$ is the measure of the leg <u>adjacent</u> to $\angle A$ divided by the measure of the leg <u>opposite</u> $\angle A$.
3. In <u>any</u> triangle, the sum of the squares of the measures of the legs equals the square of the measure of the hypotenuse.
4. An angle of <u>elevation</u> is the angle between the line of sight and the horizontal when an observer looks upward.
5. The geometric mean between two numbers is the positive square root of their <u>product</u>.
6. In a <u>30°-60°-90°</u> triangle, two of the sides will have the same length.
7. Looking at a city while flying in a plane is an example that uses angle of <u>elevation</u>.

Lesson-by-Lesson Review

7-1 Geometric Mean

See pages 342–348.

Concept Summary

- The geometric mean of two numbers is the square root of their product.
- You can use the geometric mean to find the altitude of a right triangle.

Examples

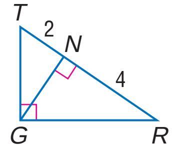
1 Find the geometric mean between 10 and 30.

$\frac{10}{x} = \frac{x}{30}$ Definition of geometric mean

$x^2 = 300$ Cross products

$x = \sqrt{300}$ or $10\sqrt{3}$ Take the square root of each side.

2 Find *NG* in $\triangle TGR$.



The measure of the altitude is the geometric mean between the measures of the two hypotenuse segments.

$\frac{TN}{GN} = \frac{GN}{RN}$ Definition of geometric mean

$\frac{2}{GN} = \frac{GN}{4}$ $TN = 2$, $RN = 4$

$8 = (GN)^2$ Cross products

$\sqrt{8}$ or $2\sqrt{2} = GN$ Take the square root of each side.

www.geometryonline.com/vocabulary_review
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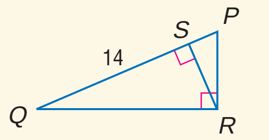
Exercises **Find the geometric mean between each pair of numbers.**
See Example 1 on page 342.

8. 4 and 16 **9.** 4 and 81 **10.** 20 and 35 **11.** 18 and 44

12. In $\triangle PQR$, $PS = 8$, and $QS = 14$. Find RS. *See Example 2 on page 344.*



7-2 The Pythagorean Theorem and Its Converse

See pages 350–356.

Concept Summary

- The Pythagorean Theorem can be used to find the measures of the sides of a right triangle.
- If the measures of the sides of a triangle form a Pythagorean triple, then the triangle is a right triangle.

Example

Find k.

$a^2 + (LK)^2 = (JL)^2$ Pythagorean Theorem

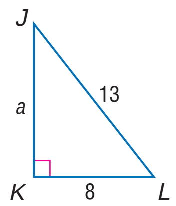
$a^2 + 8^2 = 13^2$ $LK = 8$ and $JL = 13$

$a^2 + 64 = 169$ Simplify.

$a^2 = 105$ Subtract 64 from each side.

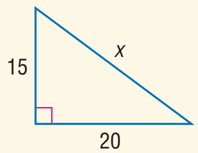
$a = \sqrt{105}$ Take the square root of each side.

$a \approx 10.2$ Use a calculator.



Exercises **Find x.** *See Example 2 on page 351.*

13.



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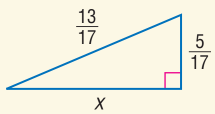

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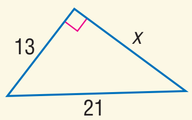


7-3 Special Right Triangles

See pages 357–363.

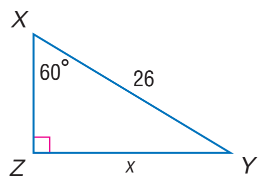
Concept Summary

- The measure of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of the legs of the triangle. The measures of the sides are x, x, and $x\sqrt{2}$.
- In a 30°-60°-90° triangle, the measures of the sides are x, $x\sqrt{3}$, and $2x$.

Examples

1 **Find x.**

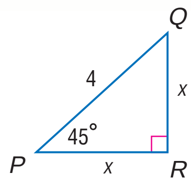
The measure of the shorter leg $\overline{XZ}$ of $\triangle XYZ$ is half the measure of the hypotenuse $\overline{XY}$. Therefore, $XZ = \frac{1}{2}(26)$ or 13. The measure of the longer leg is $\sqrt{3}$ times the measure of the shorter leg. So, $x = 13\sqrt{3}$.



2 **Find x.**

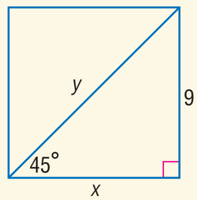
The measure of the hypotenuse of a 45°-45°-90° triangle is $\sqrt{2}$ times the length of a leg of the triangle.

$$x\sqrt{2} = 4$$
$$x = \frac{4}{\sqrt{2}}$$
$$x = \frac{4}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} \text{ or } 2\sqrt{2}$$

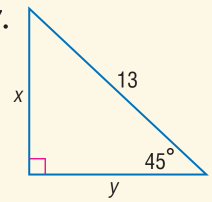


Exercises **Find x and y.** *See Examples 1 and 3 on pages 358 and 359.*

16.



17.



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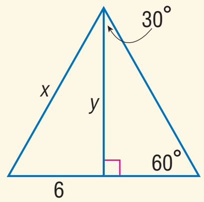
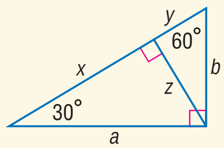


For Exercises 19 and 20, use the figure at the right.
See Example 3 on page 359.

19. If $y = 18$, find z and a.

20. If $x = 14$, find a, z, b, and y.



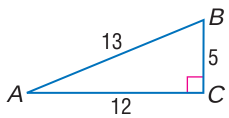
7-4 Trigonometry

See pages 364–370.

Concept Summary

- Trigonometric ratios can be used to find measures in right triangles.

Example **Find sin A, cos A, and tan A. Express each ratio as a fraction and as a decimal.**

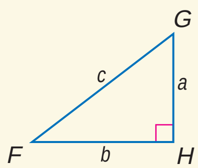


$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}} = \frac{BC}{AB} = \frac{5}{13} \text{ or about } 0.38$$

$$\cos A = \frac{\text{adjacent leg}}{\text{hypotenuse}} = \frac{AC}{AB} = \frac{12}{13} \text{ or about } 0.92$$

$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}} = \frac{BC}{AC} = \frac{5}{12} \text{ or about } 0.42$$

Exercises **Use $\triangle FGH$ to find sin F, cos F, tan F, sin G, cos G, and tan G. Express each ratio as a fraction and as a decimal to the nearest hundredth.** *See Example 1 on page 365.*



21. $a = 9, b = 12, c = 15$ **22.** $a = 7, b = 24, c = 25$

Find the measure of each angle to the nearest tenth of a degree.
See Example 4 on pages 366 and 367.

23. $\sin P = 0.4522$ **24.** $\cos Q = 0.1673$ **25.** $\tan R = 0.9324$

7-5 Angles of Elevation and Depression

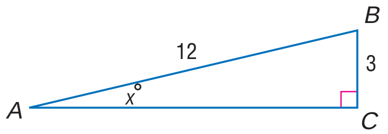
See pages 371–376.

Concept Summary

- Trigonometry can be used to solve problems related to angles of elevation and depression.

Example **A store has a ramp near its front entrance. The ramp measures 12 feet, and has a height of 3 feet. What is the angle of elevation?**

Make a drawing.



Let x represent $m\angle BAC$.

$\sin x° = \frac{BC}{AB}$ — $\sin x = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin x° = \frac{3}{12}$ — $BC = 3$ and $AB = 12$

$x = \sin^{-1}\left(\frac{3}{12}\right)$ — Find the inverse.

$x \approx 14.5$ — Use a calculator.

The angle of elevation for the ramp is about 14.5°.

Exercises **Determine the angles of elevation or depression in each situation.**
See Examples 1 and 2 on pages 371 and 372.

26. An airplane must clear a 60-foot pole at the end of a runway 500 yards long.

27. An escalator descends 100 feet for each horizontal distance of 240 feet.

28. A hot-air balloon descends 50 feet for every 1000 feet traveled.

29. **DAYLIGHT** At a certain time of the day, the angle of elevation of the sun is 44°. Find the length of a shadow cast by a building that is 30 yards high.

30. **RAILROADS** A railroad track rises 30 feet for every 400 feet of track. What is the measure of the angle of elevation of the track?

7-6 The Law of Sines

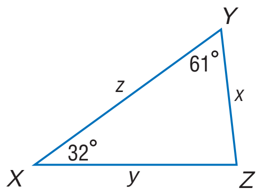
See pages 377–383.

Concept Summary

- To find the measures of a triangle by using the Law of Sines, you must either know the measures of two angles and any side (AAS or ASA), or two sides and an angle opposite one of these sides (SSA) of the triangle.
- To solve a triangle means to find the measures of all sides and angles.

Example **Solve $\triangle XYZ$ if $m\angle X = 32$, $m\angle Y = 61$, and $y = 15$. Round angle measures to the nearest degree and side measures to the nearest tenth.**

Find the measure of $\angle Z$.



$m\angle X + m\angle Y + m\angle Z = 180$ — Angle Sum Theorem

$32 + 61 + m\angle Z = 180$ — $m\angle X = 32$ and $m\angle Y = 61$

$93 + m\angle Z = 180$ — Add.

$m\angle Z = 87$ — Subtract 93 from each side.

(continued on the next page)

Chapter 7 For More ...

- Extra Practice, see pages 766–768.
- Mixed Problem Solving, see page 788.

Since we know $m\angle Y$ and y, use proportions involving sin Y and y.

To find x:		To find z:
$\frac{\sin Y}{y} = \frac{\sin X}{x}$	Law of Sines	$\frac{\sin Y}{y} = \frac{\sin Z}{z}$
$\frac{\sin 61°}{15} = \frac{\sin 32°}{x}$	Substitute.	$\frac{\sin 61°}{15} = \frac{\sin 87°}{z}$
$x \sin 61° = 15 \sin 32°$	Cross products	$z \sin 61° = 15 \sin 87°$
$x = \frac{15 \sin 32°}{\sin 61°}$	Divide.	$z = \frac{15 \sin 87°}{\sin 61°}$
$x \approx 9.1$	Use a calculator.	$z \approx 17.1$

Exercises **Find each measure using the given measures of $\triangle FGH$. Round angle measures to the nearest degree and side measures to the nearest tenth.**
See Example 1 on page 378.

31. Find f if $g = 16$, $m\angle G = 48$, and $m\angle F = 82$.

32. Find $m\angle H$ if $h = 10.5$, $g = 13$, and $m\angle G = 65$.

Solve each $\triangle ABC$ described below. Round angle measures to the nearest degree and side measures to the nearest tenth. *See Example 2 on pages 378 and 379.*

33. $a = 15, b = 11, m\angle A = 64$

34. $c = 12, m\angle C = 67, m\angle A = 55$

35. $m\angle A = 29, a = 4.8, b = 8.7$

36. $m\angle A = 29, m\angle B = 64, b = 18.5$

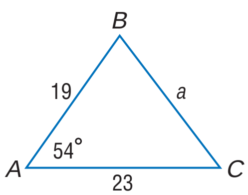
7-7 The Law of Cosines

See pages 385–390.

Concept Summary

- The Law of Cosines can be used to solve triangles when you know the measures of two sides and the included angle (SAS) or the measures of the three sides (SSS).

Example **Find a if $b = 23$, $c = 19$, and $m\angle A = 54$.**



Since the measures of two sides and the included angle are known, use the Law of Cosines.

$a^2 = b + c^2 - 2bc \cos A$	Law of Cosines
$a^2 = 23^2 + 19^2 - 2(23)(19) \cos 54°$	$b = 23$, $c = 19$, and $m\angle A = 54$
$a = \sqrt{23^2 + 19^2 - 2(23)(19) \cos 54°}$	Take the square root of each side.
$a \approx 19.4$	Use a calculator.

Exercises **In $\triangle XYZ$, given the following measures, find the measure of the missing side.** *See Example 1 on page 385.*

37. $x = 7.6, y = 5.4, m\angle Z = 51$

38. $x = 21, m\angle Y = 73, z = 16$

Solve each triangle using the given information. Round angle measure to the nearest degree and side measure to the nearest tenth.
See Example 3 on pages 386 and 387.

39. $c = 18, b = 13, m\angle A = 64$

40. $b = 5.2, m\angle C = 53, c = 6.7$

Chapter 7 Practice Test

Vocabulary and Concepts

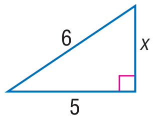
1. **State** the Law of Cosines for $\triangle ABC$ used to find $m\angle C$.
2. **Determine** whether the geometric mean of two perfect squares will always be rational. Explain.
3. **Give** an example of side measures of a 30°-60°-90° triangle.

Skills and Applications

Find the geometric mean between each pair of numbers.

4. 7 and 63
5. 6 and 24
6. 10 and 50

Find the missing measures.

7.


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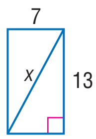

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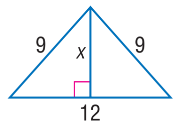

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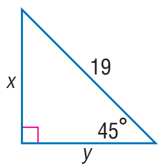

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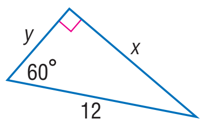

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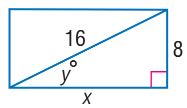
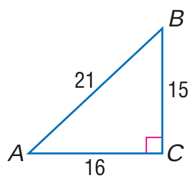


Use the figure to find each trigonometric ratio. Express answers as a fraction.



13. $\cos B$
14. $\tan A$
15. $\sin A$

Find each measure using the given measures from $\triangle FGH$. Round to the nearest tenth.

16. Find g if $m\angle F = 59$, $f = 13$, and $m\angle G = 71$.
17. Find $m\angle H$ if $m\angle F = 52$, $f = 10$, and $h = 12.5$.
18. Find f if $g = 15$, $h = 13$, and $m\angle F = 48$.
19. Find h if $f = 13.7$, $g = 16.8$, and $m\angle H = 71$.

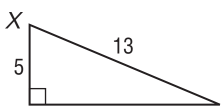
Solve each triangle. Round each angle measure to the nearest degree and each side measure to the nearest tenth.

20. $a = 15$, $b = 17$, $m\angle C = 45$
21. $a = 12.2$, $b = 10.9$, $m\angle B = 48$
22. $a = 19$, $b = 23.2$, $c = 21$

23. **TRAVEL** From an airplane, Janara looked down to see a city. If she looked down at an angle of 9° and the airplane was half a mile above the ground, what was the horizontal distance to the city?

24. **CIVIL ENGINEERING** A section of freeway has a steady incline of 10°. If the horizontal distance from the beginning of the incline to the end is 5 miles, how high does the incline reach?

25. **STANDARDIZED TEST PRACTICE** Find tan X.



(A) $\frac{5}{12}$ (B) $\frac{12}{13}$ (C) $\frac{17}{12}$ (D) $\frac{12}{5}$

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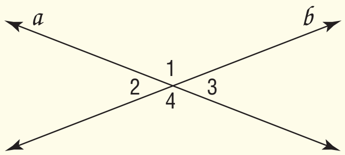
Chapter 7 Standardized Test Practice

Part 1 Multiple Choice

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

1. If $\angle 4$ and $\angle 3$ are supplementary, which reason could you use as the first step in proving that $\angle 1$ and $\angle 2$ are supplementary? (Lesson 2-7)



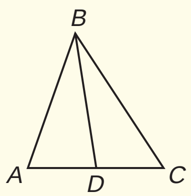
Ⓐ Definition of similar angles

Ⓑ Definition of perpendicular lines

Ⓒ Definition of a vertical angle

Ⓓ Division Property

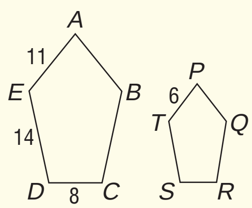
2. In $\triangle ABC$, $\overline{BD}$ is a median. If $AD = 3x + 5$ and $CD = 5x - 1$, find AC. (Lesson 5-1)



Ⓐ 3 Ⓑ 11

Ⓒ 14 Ⓓ 28

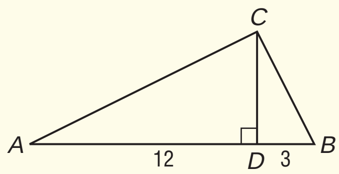
3. If pentagons $ABCDE$ and $PQRST$ are similar, find SR. (Lesson 6-2)



Ⓐ $14\frac{2}{3}$ Ⓑ $4\frac{4}{11}$

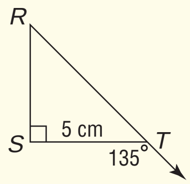
Ⓒ 3 Ⓓ $1\frac{5}{6}$

4. In $\triangle ABC$, $\overline{CD}$ is an altitude and $m\angle ACB = 90°$. If $AD = 12$ and $BD = 3$, find AC. (Lesson 7-1)



Ⓐ 6.5 Ⓑ 9.0 Ⓒ 13.4 Ⓓ 15.0

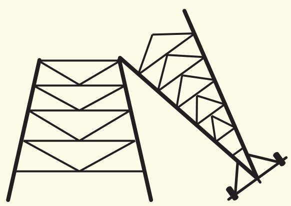
5. What is the length of $\overline{RT}$? (Lesson 7-3)



Ⓐ 5 cm Ⓑ $5\sqrt{2}$ cm

Ⓒ $5\sqrt{3}$ cm Ⓓ 10 cm

6. An earthquake damaged a tower that carries power lines. As a result, the top of the tower broke off at a point 60 feet above the base. If the fallen portion of the tower made a 36° angle with the ground, what was the approximate height of the original tower? (Lesson 7-4)



Ⓐ 35 ft Ⓑ 95 ft

Ⓒ 102 ft Ⓓ 162 ft

7. Miraku drew a map showing Regina's house, Steve's apartment, and Trina's workplace. The three locations formed $\triangle RST$, where $m\angle R = 34$, $r = 14$, and $s = 21$. What is $m\angle S$? (Lesson 7-6)

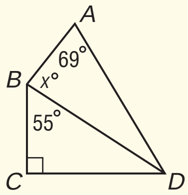
Ⓐ 15 Ⓑ 22 Ⓒ 57 Ⓓ 84

Aligned and verified by The Princeton Review

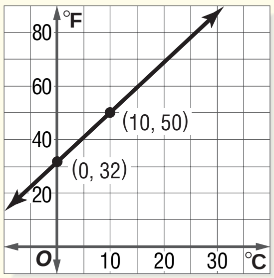
Part 2 Short Response/Grid In

Record your answers on the answer sheet provided by your teacher or on a sheet of paper.

8. Find $m\angle ABC$ if $m\angle CDA = 61$. (Lesson 1-6)



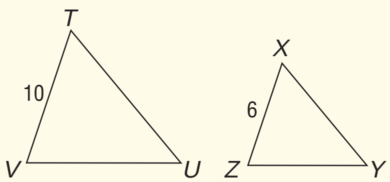
For Questions 9 and 10, refer to the graph.



9. At the International Science Fair, a Canadian student recorded temperatures in degrees Celsius. A student from the United States recorded the same temperatures in degrees Fahrenheit. They used their data to plot a graph of Celsius versus Fahrenheit. What is the slope of their graph? (Lesson 3-3)

10. Students used the equation of the line for the temperature graph of Celsius versus Fahrenheit to convert degrees Celsius to degrees Fahrenheit. If the line goes through points (0, 32) and (10, 50), what equation can the students use to convert degrees Celsius to degrees Fahrenheit? (Lesson 3-4)

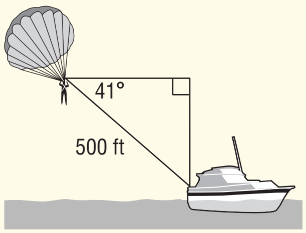
11. $\triangle TUV$ and $\triangle XYZ$ are similar. Calculate the ratio $\frac{YZ}{UV}$. (Lesson 6-3)



The Princeton Review Test-Taking Tip

Questions 6, 7, and 12 If a standardized test question involves trigonometric functions, draw a diagram that represents the problem and use a calculator (if allowed) or the table of trigonometric relationships provided with the test to help you find the answer.

12. Dee is parasailing at the ocean. The angle of depression from her line of sight to the boat is 41°. If the cable attaching Dee to the boat is 500 feet long, how many feet is Dee above the water? (Lesson 7-5)



Part 3 Open Ended

Record your answers on a sheet of paper. Show your work.

13. Toby, Rani, and Sasha are practicing for a double Dutch rope-jumping tournament. Toby and Rani are standing at points T and R and are turning the ropes. Sasha is standing at S, equidistant from both Toby and Rani. Sasha will jump into the middle of the turning rope to point X. Prove that when Sasha jumps into the rope, she will be at the midpoint between Toby and Rani. (Lessons 4-5 and 4-6)